Confusion and Inflation*

Xiaojie Liu† Dalton Rongxuan Zhang‡

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Abstract

High-inflation episodes often feature confusion about the source of aggregate shocks. We develop a theory of this confusion. Agents observe noisy signals about output and inflation and may confuse demand and supply shocks. When inflation is more informative, agents tend to interpret high inflation as signal of both adverse supply and strong demand. This confusion amplifies the inflation response to true supply shocks and dampens the expected-output response to true demand shocks. Estimating the model before and after 1990 reveals a shift from an inflation-signal-dominant to an output-signal-dominant regime, helping explain stronger pre-1990 inflation responses to supply shocks, the flattening of the Phillips curve, and the supply-side view in expectations.

1 Introduction

U.S. CPI inflation rose from 1.3 percent in 2020Q4 to 9.0 percent in 2022Q2, the sharpest two-year increase since 1979, despite three decades of evidence that the Phillips curve had flattened. The surge was accompanied by pervasive uncertainty among firms, households, economists, and policymakers about its underlying source. Two prominent diagnoses

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†Kellogg School of Management, Northwestern University. Email: xiaojie.liu@kellogg.northwestern.edu

‡Department of Economics, Northwestern University. Email: r.zhang@u.northwestern.edu

illustrate the debate: some attributed the rise mainly to *excess demand*, while others (Bernanke and Blanchard, 2023) emphasized the role of *supply-chain disruptions*.[1] Lawrence Summers captured this confusion succinctly: "*I'm not sure we would have today's inflation without the pandemic, and I'm not sure we would have it without the overwhelming stimulus we applied.*" Such confusion is not unique to the pandemic era. During the Great Inflation of the 1970s, Federal Reserve Chair Arthur Burns attributed rising prices to "*special factors*" such as oil, harvest failures, and the removal of price controls, whereas later work emphasized accommodative monetary policy (Clarida et al., 2000). In both episodes, inflation surged precisely when observers disagreed over whether the underlying force was demand or supply.

This paper argues that confusion between aggregate shocks is central to inflation dynamics. Agents learn from aggregates such as unemployment, CPI inflation, and salient prices for food and oil. But the same observed movement can arise from different structural shocks. Inflation, for example, may rise because demand is strong or because supply is impaired. It is therefore natural for agents to confuse shocks across sources. This differs from canonical information-friction models, where agents are typically uncertain about the magnitude of a single shock. Here, agents are uncertain about both the size and the source of the shock they are observing. Confusion can amplify responses, rather than the standard dampening effect. Quantitatively, we find confusion helps explain stronger pre-1990 inflation responses to supply shocks, the flattening of the Phillips curve, and the supply-side view in expectations.

We first develop a theory of confusion between demand and supply shocks. Agents do not observe structural shocks directly; instead, they infer them from noisy endogenous outcomes, such as output and inflation. Output loads positively on both demand and supply, while inflation loads positively on demand but negatively on supply. This creates an identification problem: high output may reflect either strong demand or strong supply, and high inflation may reflect either strong demand or weak supply. We define *confusion* as the cross-loading of beliefs about one shock on the other. Its sign depends on relative

[1] Prominent observers revised their views as new evidence accumulated. Paul Krugman later described his early-2021 "*Team Transitory*" view of the $1.9 trillion American Rescue Plan as "*a very bad call*." Treasury Secretary Janet Yellen likewise stated that she had been "*wrong about the path inflation would take*," pointing to unanticipated energy shocks and supply bottlenecks she "*didn't fully understand*."

signal informativeness. When inflation is more informative, agents interpret demand-driven inflation partly as adverse supply, generating *negative confusion*. When output is more informative, agents interpret supply-driven output partly as demand, generating *positive confusion*. The magnitude of confusion is non-monotone in signal precision: it vanishes when both signals are perfectly precise or completely uninformative, and it is largest when agents learn from only one informative signal.

We then show that confusion has distinctive implications for higher-order beliefs. Unlike standard dispersed-information models, where higher-order beliefs become increasingly anchored to the prior and therefore respond less to aggregate shocks, higher-order confusion can rise with the order of beliefs. Suppose, for instance, that inflation is the more informative signal. Each agent then expects others to rely relatively more on inflation than on output to learn about the economy, so higher-order beliefs become increasingly centered around the inflation signal. As a result, agents can become more confused about how others interpret demand and supply than they are about the shocks themselves.

Finally, we characterize when learning over time corrects confusion. Learning eliminates confusion only when later observations give agents new information that helps distinguish demand from supply. With two informative signals, agents can gradually separate demand from supply. With only one informative signal, later observations reveal the same mixture of shocks, so confusion persists. This logic matters for inflation dynamics: when one signal is much more precise than the other, confusion can persist.

We then embed confusion in a stylized New Keynesian model. With confusion, shock responses are not separable: beliefs about demand and supply must be solved jointly. When inflation is the more informative signal, firms interpret a positive supply shock partly as weak demand. This lowers expected inflation and perceived real marginal cost, amplifying the disinflationary effect of supply shocks. The same information structure also shapes expectations about demand shocks. Firms interpret demand-driven inflation partly as adverse supply, so expected output can respond little, or even negatively, to demand shocks despite a rise in actual output. Thus, expectations of output may appear supply-driven even when realized output fluctuations are mostly demand-driven, consistent with survey

evidence (Candia et al., 2023a). In contrast, when output is more informative, the sign of confusion reverses, which can make positive supply shocks less disinflationary.

Confusion introduces a new general-equilibrium channel. In canonical incomplete-information models, information frictions mainly generate GE attenuation: GE feedback is dampened because agents place less weight on the shock and more weight on the prior in their higher-order beliefs (Angeletos and Lian, 2023). Here, confusion adds cross-shock GE feedback. It changes the *direction*, not only the *magnitude*, of equilibrium responses. Since agents place weight on the wrong shock in their beliefs, the partial-equilibrium response to one shock feeds into the general-equilibrium response to the other. Therefore, the GE feedback can be even larger than the full-information case.

We quantify the model using real-time U.S. macroeconomic data and survey expectations from the Survey of Professional Forecasters. The sample runs from 1969Q3 to 2019Q4 and is split into pre-1990 and post-1990 periods. We calibrate standard preference, technology, and policy parameters, and estimate four key variance parameters: demand-shock variance, supply-shock variance, inflation-signal noise, and output-signal noise. The estimates show a sharp decline in both demand and supply volatility after 1990. More importantly, they reveal a change in the information environment. Before 1990, the inflation signal is much more precise than the output signal, generating an inflation-signal-dominant regime and negative confusion. After 1990, the output signal becomes relatively more informative, shifting the economy toward an output-signal-dominant regime and positive confusion.

Confusion explains why expectations move so much. We invert realized output and inflation through the estimated model to recover demand and supply shocks, then feed these shocks back through the model to generate expectations. The model matches expectation dynamics while preserving a large output response to demand shocks. This combination is difficult in canonical one-shock incomplete-information models: weak information about demand is needed to generate large output movements, but it also makes expectations move too little. Confusion resolves the tension. Agents can have limited information about demand shocks and still form volatile expectations because they misattribute movements to supply shocks, which the literature calls "supply-side view" of aggregate fluctuations.

The historical decomposition further clarifies the "supply-side view" of expectations. Before 1990, demand shocks account for about 40% of realized output fluctuations, but expected output is almost entirely supply driven. Supply shocks also explain nearly all variation in realized and expected inflation. This occurs because agents learn mainly from the precise inflation signal, and negative confusion leads them to infer adverse supply shocks, which dampens the expected-output response to demand shocks.

The impulse responses reveal that supply shocks generate stronger responses before 1990 due to stronger own-shock perception and negative confusion. Moreover, the slope of the Phillips curve is flattened by 60% after 1990. The key difference in these two samples is the sign of perceived supply: before 1990, negative confusion raises expected inflation and strengthens the inflation response; after 1990, positive confusion lowers expected inflation and weakens the inflation response.

Finally, using the pre-1990 estimates, we study a more accommodative monetary-policy rule. The counterfactual reveals an amplification loop between lenient policy and inflation through a new confusion channel. A weaker response to inflation makes a demand shock generate larger price movements; because inflation is precisely observed in the pre-1990 regime, agents infer both a larger demand shock and stronger cross-shock confusion. Both channels further raise inflation responses to both demand and supply shocks. Thus accommodative policy amplifies inflation not only directly, but also by distorting inference about the source of shocks.

Literature This paper contributes to three related literatures. First, it contributes to the literature on information frictions, monetary non-neutrality, and business-cycle dynamics (Lucas, 1972; Mankiw and Reis, 2002; Beaudry and Portier, 2006; Maćkowiak and Wiederholt, 2009; Hellwig and Venkateswaran, 2009; Angeletos and La'o, 2010; Nimark, 2014; Melosi, 2017; Rondina and Walker, 2017; Chahrour and Jurado, 2018; Angeletos and Huo, 2021; Huo and Pedroni, 2023; Pfäuti, 2023; Huo et al., 2024; Huo and Takayama, 2025). Existing work studies how noisy information attenuates responses to aggregate shocks or generates sluggish adjustment. We emphasize a different mechanism: with multiple aggregate shocks, information frictions can make agents confuse one shock for another. This cross-shock confusion changes both the magnitude and the sign of equilibrium responses.

Second, the paper contributes to the empirical literature on macroeconomic expectations (Mankiw et al., 2004; Coibion and Gorodnichenko, 2015; Malmendier and Nagel, 2016; Cavallo et al., 2017; Coibion et al., 2018b,a; Nakamura and Steinsson, 2018; Coibion et al., 2020; Bordalo et al., 2020; D'Acunto et al., 2021; Angeletos et al., 2021). Recent evidence shows that attention to inflation varies across regimes and rises when inflation is high (Weber et al., 2023). We show that the relative informativeness of inflation and output signals is crucial for how agents interpret macroeconomic fluctuations. Although the model does not endogenize information acquisition, estimating it separately across inflation regimes shows that shifts in information environment change both the sign and magnitude of confusion. This mechanism helps resolve puzzles in the survey evidence, such as the supply-side view documented by Candia et al. (2023b). Our paper is complementary to (Kamdar et al., 2018) and (Kamdar and Ray, 2024), who study the optimal attention problem and show that liquidity-constrained consumers optimally pay greater attention to supply shocks because they are more exposed to them, generating a supply-side view of the economy. We instead study how confusion about the source of aggregate shocks reshapes inflation dynamics, producing strong supply-shock inflation responses and a flatter Phillips curve.

Third, we relate to the literature on Phillips curve flattening (Stock and Watson, 2019; Jorgensen and Lansing, 2019; Negro et al., 2020; McLeay and Tenreyro, 2020; Afrouzi and Yang, 2021; Barnichon and Mesters, 2020; Hazell et al., 2022; Pfäuti, 2023). Our model provides a new explanation for the flattening of the Phillips curve through confusion. Before 1990, negative confusion raises the inflation response to demand shocks because firms partly interpret them as adverse supply shocks and supply shocks drive large inflation response. The quantification also shows that lenient monetary policy amplifies negative confusion, further steepening the Phillips curve.

Outline. Section 2 develops a general theory of confusion in partial equilibrium. Section 3 embeds the mechanism in a simple New Keynesian model and characterizes its general-equilibrium effects. Section 4 presents the quantitative model. Section 5 brings the model to the data and reports the main quantitative results. Section 6 concludes.

2 A Theory of Confusion

We begin with a signal-extraction problem that isolates confusion in partial equilibrium. When agents infer structural shocks from endogenous aggregates, their beliefs may load on the wrong shock. We call this cross-loading *confusion*. This section defines confusion, characterizes its sign and magnitude, and studies how it propagates through higher-order beliefs and learning dynamics. The next section embeds this inference problem in a New Keynesian model, where beliefs affect output and inflation and therefore feed back to the signals agents observe.

Agents do not observe aggregate shocks directly. Demand and supply shocks have distinct implications for output and inflation: in the standard New Keynesian framework, a positive demand shock raises both output and inflation, whereas a positive supply shock raises output but lowers inflation. To isolate the source of confusion, we focus on two signals: an output signal and an inflation signal[2],

$$x_{it}^{y} = \omega_{ys} a_t + \omega_{yd} \beta_t + \epsilon_{it}^{y}, \quad x_{it}^{\pi} = -\omega_{\pi s} a_t + \omega_{\pi d} \beta_t + \epsilon_{it}^{\pi}, \tag{1}$$

where all ω's are strictly positive. The coefficients ω_{ys} and $\omega_{\pi s}$ are the responses of output and inflation to a unit supply shock, while ω_{yd} and $\omega_{\pi d}$ are their responses to a unit demand shock. The aggregate shocks follow Gaussian and have variances σ_d^2 and σ_s^2. The noise terms satisfy $\epsilon_{it}^{y} \sim \mathcal{N}(0, \sigma_y^2)$ and $\epsilon_{it}^{\pi} \sim \mathcal{N}(0, \sigma_\pi^2)$. Signal noises are independent across agents, signals, and are independent of aggregate shocks. The negative sign on $\omega_{\pi s}$ captures the disinflationary effect of a positive supply shock. It is important to notice that when both σ_y and σ_π converge to zero, agents can recover both structural shocks exactly: the two observed signals provide two independent equations for the two unknown shocks. Confusion therefore arises only when signals are noisy that agents cannot perfectly disentangle demand from supply.

This structure is the most parsimonious way to capture confusion between aggregate demand and supply shocks. A high output signal can reflect either low cost or strong

[2]Agents may observe additional signals, such as unemployment, wages, or sectoral output. In this two-shock environment, however, such signals amount to different rotations of output and inflation. We therefore work with the minimal two-signal structure.

demand. A high inflation signal can reflect either strong demand or high cost. Thus, the same observed aggregate movement may be attributed to different structural sources. At this stage, it is immaterial whether signal loadings are endogenous impulse responses or exogenously given loadings; the goal is only to characterize belief formation under this signal structure.

Proposition 1 (Confusion in Belief Formation)**.** *Given the signal structure in* (1)*, the projection from aggregate shocks to average beliefs is*

$$\begin{pmatrix} \bar{E}\beta \\ \bar{E}a \end{pmatrix} = \mathbf{P} \begin{pmatrix} \beta \\ a \end{pmatrix} = \begin{pmatrix} D_d & C \\ C & D_s \end{pmatrix} \begin{pmatrix} \beta \\ a \end{pmatrix},$$

where C is the confusion term and $D_d, D_s < 1$. Moreover,

$$C \propto \frac{1}{\sigma_y^2}\omega_{yd}\omega_{ys} - \frac{1}{\sigma_\pi^2}\omega_{\pi d}\omega_{\pi s}. \tag{2}$$

where the omitted proportionality factor is strictly positive.

This proposition establishes that, under the signal structure described in (1), firms' confusion C is generally nonzero. Importantly, the sign of confusion is determined by the relative informativeness of both signals, given by the product of signal loadings weighted by the signal noise. Intuitively, if inflation responds more to shocks (i.e., if $\omega_{\pi d}, \omega_{\pi s}$ are large), inflation contains more information and agents will learn more from it. Since an increase in inflation can be due to either a positive demand shock or a negative supply shock, learning more from inflation leads agents to confuse a *negative* supply shock with a *positive* demand shock, i.e., $C < 0$.

Conversely, if output is more responsive to shocks (i.e., if ω_{yd}, ω_{ys} are large), agents will extract more information from output. In this case, an increase in output can be due to either a positive demand shock or a positive supply shock, so learning more from output leads agents to confuse a *positive* supply shock with a *positive* demand shock, i.e., $C > 0$.

2.1 The Magnitude of Confusion

Confusion is strongest neither when signals are perfectly precise nor when they are entirely uninformative. It is strongest when signals are informative enough to move beliefs but insufficient to identify the two shocks separately.

Proposition 2 (Magnitude of Confusion)**.** *Fix the relative noise ratio $\rho = \sigma_y^2/\sigma_\pi^2$ and parameterize $\sigma_y^2 = \rho\tau$, $\sigma_\pi^2 = \tau$. Define $\rho^* = \frac{\omega_{yd}\omega_{ys}}{\omega_{\pi d}\omega_{\pi s}}$. Then if $\rho < \rho^*$, then $C > 0$ and first increases then decreases in τ. If $\rho > \rho^*$, then $C < 0$ and first decreases then increases in τ. In addition,*

$$\lim_{\tau\to 0} C = \lim_{\tau\to\infty} C = 0.$$

Moreover, holding σ_y fixed, C increases with σ_π; and holding σ_π fixed, C decreases with σ_y.

When both signals become perfectly precise, agents identify demand and supply shocks exactly, so confusion vanishes. When both signals become infinitely noisy, agents extract no useful information, and the entire projection matrix converges to zero. Between these limits, the magnitude of confusion is non-monotonic in signal precision: it peaks at an intermediate level of precision while keeping the ratio of the variances of noise the same.

In the second part, we consider a different form of monotone comparative statics which follows directly from the sign formula (2). Increasing σ_π makes inflation less informative relative to output, moving beliefs toward output-based inference and increasing C. Increasing σ_y makes output less informative relative to inflation, moving beliefs toward inflation-based inference and decreasing C.

Corollary 1 (Maximal Confusion)**.** *The maximal positive confusion obtains when output is perfectly precise and inflation is uninformative:*

$$C_{\max} = \frac{\sigma_d\sigma_s\omega_{yd}\omega_{ys}}{\sigma_d^2\omega_{yd}^2 + \sigma_s^2\omega_{ys}^2}.$$

The maximal negative confusion obtains when inflation is perfectly precise and output is uninformative:

$$C_{\min} = -\frac{\sigma_d\sigma_s\omega_{\pi d}\omega_{\pi s}}{\sigma_d^2\omega_{\pi d}^2 + \sigma_s^2\omega_{\pi s}^2}.$$

In the symmetric case, with $\sigma_s = \sigma_d$ and unit loadings,

$$C_{\max} = -C_{\min} = \frac{1}{2}, \qquad \mathbf{P} = \begin{pmatrix} 1/2 & \pm 1/2 \\ \pm 1/2 & 1/2 \end{pmatrix}.$$

Corollary 1 sharpens the intuition of Proposition 2 by identifying the boundary cases where confusion attains its maximal magnitude. It shows that maximal confusion occurs when agents learn from only one informative direction. If output is the only useful signal, agents observe the sum of demand and supply movements and attribute half of the variation to the wrong source in the symmetric case. If inflation is the only useful signal, agents observe the difference between demand and supply movements, producing confusion with the opposite sign.

2.2 Higher-Order Confusion

We next study higher-order confusion: agents' beliefs about other agents' confusion. We follow the tradition of the literature on higher-order beliefs (Morris and Shin, 2002) and (Angeletos and Lian, 2023). In our setting with multiple shocks, this requires separating the direction of confusion from its strength in higher-order beliefs.

Lemma 1 (Spectral Representation of Confusion)**.** *The projection matrix* $\mathbf{P}$ *is symmetric and positive semidefinite. Let*

$$\mathbf{P} = \lambda_1 v_1 v_1^\top + \lambda_2 v_2 v_2^\top, \qquad 0 \le \lambda_2 \le \lambda_1 \le 1.$$

Suppose

$$v_1 v_1^\top = \begin{pmatrix} x & \mu \\ \mu & z \end{pmatrix}, \qquad v_2 v_2^\top = \begin{pmatrix} z & -\mu \\ -\mu & x \end{pmatrix}.$$

The term μ is the eigen-confusion*. Then*

$$C = (\lambda_1 - \lambda_2)\mu.$$

For fixed ω and shock process, μ is constant for a fixed noise ratio σ_y/σ_π.

This representation separates the *direction* of confusion from its *strength*. The eigen-confusion μ determines the direction of the rotation of the projection matrix $\mathbf{P}$. The eigenvalue gap $\lambda_1 - \lambda_2$ determines how strongly information loads on that direction. For given signal weights and shock processes, the direction μ is pinned down entirely by the relative noise ratio of the two signals. If both signals become noisier while their relative precision is unchanged, the direction of confusion remains fixed and only the eigenvalue gap changes. This decomposition is useful for higher-order beliefs because belief iteration changes eigenvalues but not eigenvectors.

Proposition 3 (Higher-Order Confusion)**.** *Let $s = (\beta, a)^\top$, define $\bar{E}^1 s = \bar{E}s$, and recursively define $\bar{E}^h s = \bar{E}(\bar{E}^{h-1}s)$. Then*

$$\bar{E}^h s = \mathbf{P}^h s.$$

The associated higher-order confusion is

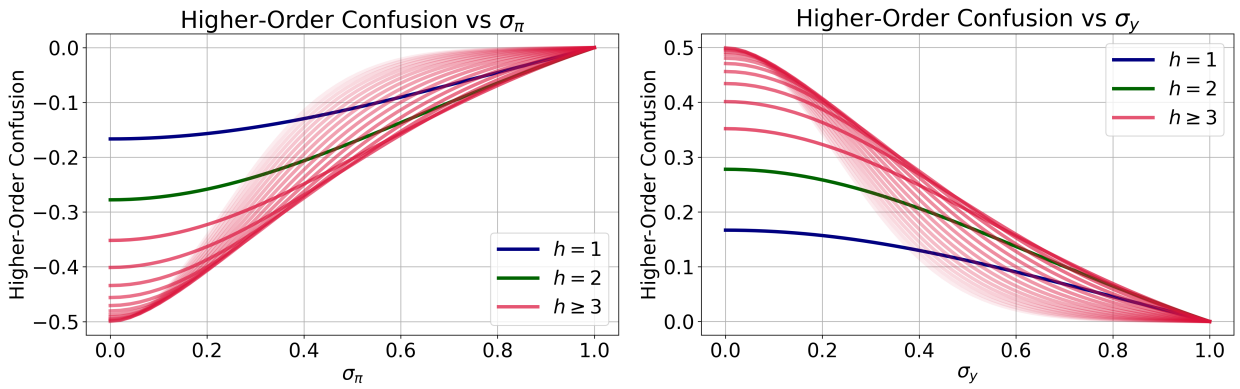
$$C_h = (\lambda_1^h - \lambda_2^h)\mu.$$

For given ω, shock process, and finite signal noises, $|C_h|$ is generally non-monotone in the order of beliefs: it first increases and then decreases with h.

The higher-order beliefs differ sharply from canonical incomplete-information models. There, higher-order beliefs become increasingly anchored to the prior and therefore respond less to aggregate shocks. Here, agents may become more confused about how others interpret demand and supply than they are about the shocks themselves. The reason is relative signal strength. As belief order increases, agents put less weight on both signals in absolute terms, but the less informative signal is discounted faster. Beliefs therefore become increasingly tilted toward the more informative signal, widening the effective eigenvalue gap and strengthening confusion. At sufficiently high orders, however, the overall loss of information dominates: both eigenvalue components decay, and confusion eventually dissipates.

Corollary 2 (Public-Signal Limit)**.** *In the limiting case in which one signal becomes perfectly precise, higher-order confusion becomes monotone. If $\sigma_y \to 0$ or $\sigma_\pi \to 0$, then $|C_h|$ increases with h.*

Figure 1: Higher-Order Confusion



Moreover,

$$\lim_{h\to\infty,\ \sigma_y\to 0} C_h = C_{\max}, \qquad \lim_{h\to\infty,\ \sigma_\pi\to 0} C_h = C_{\min}.$$

Corollary 2 shows that the non-monotonicity in Proposition 3 disappears when one signal becomes perfectly precise. In that limit, the dominant eigenvalue converges to one. The signal is effectively public, so the dominant eigenvalue of higher-order beliefs no longer decays. As a result, higher-order confusion persists and converges to the maximal confusion associated with the single informative direction.

Figure 1 illustrates the behavior of higher-order confusion. The left panel varies the noise in the inflation signal and fixes other parameters. The right panel varies the noise in the output signal. In both cases, any order of confusion is largest when one signal is nearly perfectly precise and declines as that signal becomes noisier, consistent with Proposition 2. The figure further shows that higher-order confusion is generally hump-shaped in belief order, as in Proposition 3. Moreover, higher-order confusion can be much larger than first-order confusion, especially when one signal is more informative than the other.

2.3 Dynamic Learning and Confusion

We first consider a symmetric shock process, in which demand and supply shocks have the same persistence and innovation variance. This case isolates the role of signal dimensionality: dynamics alone do not help agents distinguish demand from supply.

Learning corrects confusion only when both signals are informative. We then use simulations to show how the result changes when the two shocks have different persistence.

Let shocks follow $s_t = \rho s_{t-1} + u_t$, where $u_t \sim \mathcal{N}(0, \sigma^2 I_2)$ and $s_t = (\beta_t, a_t)^\top$. Each period, agents observe signals as in (1). Let K denote the Kalman gain. The standard Kalman filter implies

$$\bar{E}s_t = (I - KH)F\bar{E}s_{t-1} + KHs_t; \tag{3}$$

where

$$KH = \begin{pmatrix} D_d & C \\ C & D_s \end{pmatrix}; \qquad H = \begin{pmatrix} \omega_{yd} & \omega_{ys} \\ \omega_{\pi d} & -\omega_{\pi s} \end{pmatrix}; \qquad F = \rho I_2.$$

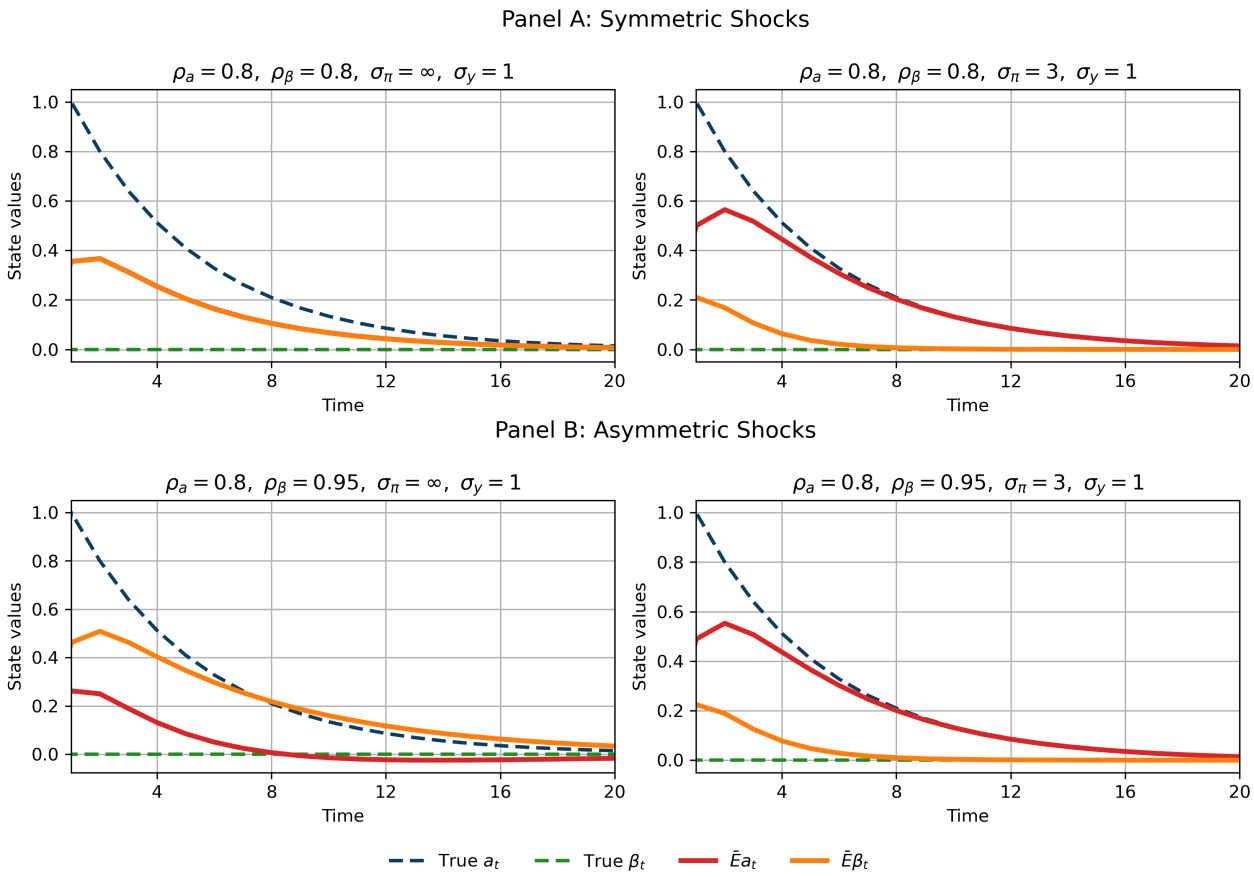
Proposition 4 (Belief Dynamics)**.** *Average beliefs evolve as*

$$\bar{E}\beta_t = \rho(1 - D_d)\bar{E}\beta_{t-1} + D_d\beta_t + C\left(a_t - \rho\bar{E}a_{t-1}\right),$$
$$\bar{E}a_t = \rho(1 - D_s)\bar{E}a_{t-1} + D_s a_t + C\left(\beta_t - \rho\bar{E}\beta_{t-1}\right).$$

Proposition 4 extends the static projection in Proposition 1 to a dynamic learning environment. The belief about each shock contains three components. The first component is the carryover of the agent's own prior belief. For example, past beliefs about demand enter current demand beliefs through $\rho(1 - D_d)\bar{E}\beta_{t-1}$. This is the standard Kalman-filter updating term. The second component is the contemporaneous signal extraction from the true shock itself. The terms $D_d\beta_t$ and $D_s a_t$ measure how much current beliefs load on the current realization of the corresponding structural shock.

The third component is confusion. In the static model, beliefs about demand load directly on the supply shock through Ca_t, and beliefs about supply load directly on the demand shock through $C\beta_t$. In the dynamic model, however, agents do not confuse the entire level of the other shock. They confuse only the part of the other shock that is not already predicted by their previous belief. Thus demand beliefs load on $C\left(a_t - \rho\bar{E}a_{t-1}\right)$, and supply beliefs load on $C\left(\beta_t - \rho\bar{E}\beta_{t-1}\right)$. Therefore, learning corrects confusion because agents use their past estimate of the other shock to undo part of the current cross-loading. This is why confusion can fade over time when both signals are informative.

Figure 2: Belief Dynamics



Notes: Panel A shows belief dynamics and true shocks when both shocks have persistence 0.8 and unit variance. Panel B shows the case with $\rho_\beta = 0.95$ and $\rho_a = 0.8$. The left column makes the inflation signal uninformative. The right column makes both signals informative, with output more informative than inflation.

The self-correction mechanism fails when learning is effectively one-dimensional. If only one signal is informative, agents never observe the other signal. Then past beliefs cannot be used to disentangle the two structural shocks; they only propagate the same mistaken direction forward. In that case, confusion persists rather than being corrected.

Corollary 3 (Permanent Confusion)**.** *If the inflation signal is uninformative, then*

$$\bar{E}\beta_t = \bar{E}a_t = \frac{1}{2}\frac{\alpha_1}{1-\rho(1-\alpha_1)L}(\beta_t + a_t).$$

where scalar $\alpha_1 \in (0,1)$*. If the output signal is uninformative, then*

$$\bar{E}\beta_t = -\bar{E}a_t = \frac{1}{2}\frac{\alpha_1}{1-\rho(1-\alpha_1)L}(\beta_t - a_t).$$

Figure 2 illustrates the dynamic mechanism. In all panels, a unit supply shock occurs at time zero, with no subsequent shocks. Panel A shuts down the inflation signal. Beliefs about demand and supply then move together, as in Corollary 3. Panel B restores the second signal, which breaks this comovement. Contemporaneous confusion makes $\bar{E}\beta_t$ initially positive, but less so than in the one-signal case. Learning then quickly corrects the mistake: $\bar{E}\beta_t$ decays rapidly, while $\bar{E}a_t$ converges to the true supply process.

Panel B considers unequal persistence, with $\rho_d = 0.95 > \rho_a = 0.8$. With only one informative signal, confusion increases further over time—perceived demand exceeds perceived supply, and $\bar{E}a_t$ turns negative. With two informative signals (bottom–right), the results are similar to the symmetric case. When demand is *less* persistent, then even one informative signal can eventually separate the shocks, as the demand impulse responses fade quickly and the remaining variation is then attributed to supply.

In sum, one-signal learning allows confusion to persist, whereas two-signal learning corrects it. The distortion is particularly severe when agents misattribute a shock to a more persistent structural shock.

3 A New Keynesian Model with Confusion

We now ask how confusion changes inflation dynamics. We embed the signal-extraction problem from Section 2 into a simple New Keynesian model. The key departure is that firms are no longer passive econometricians: they use noisy information to set prices. This section connects the theory above to two applied lessons. First, in an inflation-signal-dominant regime, confusion can generate large inflation responses to supply shocks. Second, confusion can make expectations appear supply-driven even when realized output fluctuations are largely demand driven. These forces are disciplined quantitatively in Sections 4 and 5; here we isolate them in a simple New Keynesian environment.

The timeline is as follows: In each period, Nature first draws aggregate demand and supply shocks and idiosyncratic signals. In the morning, firms set prices. Firms set a pricing schedule conditional on the output and inflation information they will observe.[3] In the

[3]This specification is similar to the pricing schedule in Vives (2017), which in turn provides one micro-market structure for Grossman and Stiglitz (1980).

afternoon, households make labor supply and consumption-saving decisions. Firms hire labor and produce to meet demand. Finally, goods and labor market clear.

Information Structure. Firms' pricing decisions determine the inflation and output signals from which they learn. We therefore distinguish exogenous from endogenous information. With exogenous information, signal loadings are fixed; with endogenous information, equilibrium responses determine the informativeness of the signals themselves, creating a fixed point between beliefs and outcomes. We first study the exogenous case to isolate the mechanism, and then impose the endogenous-information restriction in Section 3.2.

Solving a fully dynamic general-equilibrium model with incomplete information is complicated by higher-order beliefs: firms must reason not only about the aggregate state, but also about how other firms interpret it (Townsend, 1983). To isolate the main mechanism, we impose a one-period information friction. In period t, firms observe noisy signals about the current aggregate state. From period $t+1$ onward, the aggregate shocks become publicly observed. Formally, firm i's information set at time t is

$$\mathcal{I}_{it} = \{x_{it}^{y}, x_{it}^{\pi}, a_{it}, a_{t-1}, \beta_{t-1}, \mathcal{I}_{i,t-1}\}.$$

The assumptions of one-period confusion, fully informed households, and exogenous signal will be relaxed in the quantitative model in Section 4.

Household. The representative household is infinitely lived. It consumes the final good, supplies labor, and trades a risk-free nominal bond in zero net supply. The log-linearized Euler equation and labor-supply condition are

$$y_t = y_{t+1} - \frac{1}{\sigma}(i_t - \pi_{t+1}) + \beta_t \tag{4}$$

$$\eta n_t = -\sigma y_t + w_t - p_t \tag{5}$$

Here y_t is aggregate output, which equals aggregate consumption in equilibrium; $\pi_t = p_t - p_{t-1}$ is inflation; p_t is the aggregate price index; i_t is the nominal interest rate; n_t is aggregate labor; w_t is the nominal wage; σ is the inverse intertemporal elasticity of

substitution; and η is the inverse Frisch elasticity of labor supply. The demand shock follows $\beta_t = \rho_d \beta_{t-1} + u_t^d$, where $u_t^d \sim \mathcal{N}(0, \sigma_d^2)$. An increase in β_t raises current demand.

Monetary Authority. The monetary authority follows the Taylor rule, $i_t = \phi \pi_t$.

Firms. The setup of firms draws heavily from Woodford (2003). The final consumption good is consisted of a continuum of differential goods aggregated with CES, i.e., $C_t = \int_0^1 C_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj$, implying the demand curve for each good $C_{jt} = (\frac{P_{jt}}{P_t})^{-\varepsilon} C_t$, where P_t is the price index defined as $P_t = (\int_0^1 P_{jt}^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}$. Each variety is produced by a monopolistic firm with the production function $Y_{it} = A_{it} N_{it}^{1-\alpha}$. The decreasing return to scale gives rise to strategic complementarity. Firms are subject to both aggregate and idiosyncratic productivity shocks, i.e., $a_{it} = \rho_a a_t + \epsilon_{it}$, where ϵ_{it} follows $\mathcal{N}(0, \sigma_\epsilon^2)$ and a_t follows $a_t = \rho_a a_{t-1} + \epsilon_s$ where $\epsilon_s \sim \mathcal{N}(0, \sigma_s^2)$.

Firm i sets price P_{it} to maximize profit conditional on its information set period by period. At date t, firm i's problem is,

$$\max_{P_{it}, Y_{it}, N_{it}} \mathbb{E}_{it}\left(\frac{U'(C_t)}{P_t}(P_{it} Y_{it} - W_t N_{it})\right)$$
$$\text{s.t.} \quad Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t; \quad Y_{it} = A_{it} N_{it}^{1-\alpha}$$

where $U'(C_t)$ is the marginal utility of consumption, and N_{it} is the labor hired. The log-linearized optimal pricing condition is

$$p_{it} = \mathbb{E}_{it}\left[\varphi\left(p_t + \frac{1}{\varepsilon} y_t\right) + (1-\varphi) w_t\right] - \frac{1-\varphi}{1-\alpha} a_{it}, \tag{6}$$

where

$$\varphi = \frac{\alpha/(1-\alpha)}{\alpha/(1-\alpha) + 1/\varepsilon}.$$

As in Woodford (2003), firms must forecast both the intercept of its demand curve, $p_t + \frac{1}{\varepsilon} y_t$, and the nominal wage. The intercept captures strategic complementarity: a higher aggregate price level or stronger aggregate demand raises the price a firm can charge for a given quantity sold. The nominal wage appears in expectation because firms do not know the

wage before setting prices. In the afternoon, firms will hire labor to clear the demand for their goods at the market-clearing nominal wage.

The log-linearized labor demand is $n_{it} = \frac{1}{1-\alpha}(y_{it} - a_{it})$. Integrate it out and combine it with labor supply condition in equation (5), we obtain the labor market clear condition,

$$w_t - p_t = \sigma y_t + \frac{\eta}{1-\alpha}(y_t - a_t) \tag{7}$$

Firms understand that goods and labor markets clear in equilibrium. Substituting (7) into the pricing condition (6), aggregating across firms, and normalizing $p_{t-1} = 0$, so that $\pi_t = p_t$, yields the aggregate pricing equation

$$\pi_t = \bar{\mathbb{E}}_t \left(\pi_t + \psi_y y_t\right) - \psi_a \left(\eta \bar{\mathbb{E}}_t a_t + a_t\right), \tag{8}$$

where

$$\psi_y = \frac{\varphi}{\varepsilon} + (1-\varphi)\left(\sigma + \frac{\eta}{1-\alpha}\right), \qquad \psi_a = \frac{1-\varphi}{1-\alpha}.$$

Equation (8) shows that inflation is driven by three forces: firms' expectations of other firms' prices, expected real wage, and actual aggregate productivity, which is the result of integration of individual productivity.

3.1 Confusion in General Equilibrium

Canonical macroeconomic models typically study the equilibrium responses to each aggregate shock in isolation. With confusion, however, the responses to one shock depends on the equilibrium responses to the other. The two impulse responses must therefore be solved jointly. The following proposition characterizes the joint impact responses of inflation and output to unit demand and supply shocks. Let $\tilde{z}_d$ denote the response of variable z to a unit demand shock β_t, and let $\tilde{z}_s$ denote the response to a unit supply shock a_t.

Proposition 5 (System of Equations). *The joint responses* $\{\tilde{\pi}_d, \tilde{\pi}_s, \tilde{y}_d, \tilde{y}_s\}$ *solve*

$$\begin{aligned}
\tilde{\pi}_d &= (1 - D_d)^{-1} \left[\psi_y D_d \tilde{y}_d + C\left(\tilde{\pi}_s + \psi_y \tilde{y}_s\right) - \eta \psi_a C\right], \\
\tilde{\pi}_s &= (1 - D_s)^{-1} \left[\psi_y D_s \tilde{y}_s + C\left(\tilde{\pi}_d + \psi_y \tilde{y}_d\right) - \eta \psi_a D_s - \psi_a\right], \\
\tilde{y}_d &= -\frac{\phi}{\sigma}\tilde{\pi}_d + F_d, \\
\tilde{y}_s &= -\frac{\phi}{\sigma}\tilde{\pi}_s + F_s,
\end{aligned}$$

where $F_d = \frac{\phi}{\phi - \rho_d}$, $F_s = (1 + \eta)\frac{\psi_a}{\psi_y}\rho_a \frac{\phi - 1}{\phi - \rho_a}$.

The demand-side equations have a standard interpretation. The term $-\frac{\phi}{\sigma}\tilde{\pi}_j$, for $j \in \{d, s\}$, captures the effect of the contemporaneous real interest rate on consumption. The terms F_d and F_s summarize expected future output and inflation. Because households are fully informed and know that today's shocks become common knowledge from period $t + 1$ onward, F_d and F_s are computed from the next-period full-information equilibrium. The term F_s is positive because a positive supply shock raises the real wage, which outweighs the decline in inflation.

The key departure from standard incomplete-information models is that the two impulse responses are jointly determined. When $C = 0$, the cross-shock terms vanish, the system decouples, and the model reduces to a benchmark incomplete-information economy with only expectation attenuation through D_d and D_s, as in Woodford (2003) and Angeletos and La'o (2010). When $C \neq 0$, by contrast, the pricing response to each shock inherits part of the equilibrium response to the other.

Confusion operates through two cross-shock channels. The first is an expected-inflation channel. Because prices are strategic complements, firms respond to supply-driven inflation after a demand shock, $C\tilde{\pi}_s$, and to demand-driven inflation after a supply shock, $C\tilde{\pi}_d$. The second is an expected-marginal-cost channel. After a demand shock, firms partially interpret the shock as supply-driven, generating the term $C(\psi_y \tilde{y}_s - \eta\psi_a)$. After a supply shock, firms partially interpret the shock as demand-driven, generating $C\psi_y \tilde{y}_d$.

The sign of confusion determines the direction of misinterpretation. To fix ideas, consider the inflation-dominant case, $(C < 0)$, in which a positive supply shock is partly interpreted as a negative demand shock. The output-dominant case, $(C > 0)$, reverses the signs. Since a

positive demand shock raises both inflation and output, $\tilde{\pi}_d > 0$ and $\tilde{y}_d > 0$, both cross-shock channels push prices downward, i.e., $C\tilde{\pi}_d < 0$ and $C\psi_y\tilde{y}_d < 0$. Thus confusion amplifies the disinflationary effect of a positive supply shock and, through the Euler equation, amplifies the output response. Intuitively, firms misinterpret the positive supply shock as weak demand. Instead of expecting stronger real activity, they expect lower demand and lower real marginal cost, and reduce prices further.

After a positive demand shock, the effect is not signed in general. When $C < 0$, the expected-inflation channel raises prices because $C\tilde{\pi}_s > 0$ and $\tilde{\pi}_s < 0$. The marginal-cost channel is ambiguous because it depends on supply-shock persistence and the equilibrium inflation response, as characterized in Corollary A.1. Hence confusion has a sharply signed effect after supply shocks but an ambiguous effect after demand shocks.

GE Effects and Higher-Order Confusion. We now solve the equilibrium responses in Proposition 5 and isolate the role of higher-order confusion.

Theorem 1 (Higher-Order Confusion in General Equilibrium)**.** *Let*

$$\tilde{\boldsymbol{\pi}} = \begin{pmatrix} \tilde{\pi}_d \\ \tilde{\pi}_s \end{pmatrix}, \qquad \mathbf{U} = \begin{pmatrix} U_d \\ U_s \end{pmatrix} = \begin{pmatrix} \psi_y(D_dF_d + CF_s) - \eta\psi_a C \\ \psi_y(CF_d + D_sF_s) - \eta\psi_a D_s - \psi_a \end{pmatrix}, \qquad \delta = 1 - \frac{\phi\psi_y}{\sigma}.$$

Then

$$\tilde{\boldsymbol{\pi}} = (\mathbf{I} - \delta\mathbf{P})^{-1}\mathbf{U}.$$

If $|\delta| < 1$*, then*

$$\tilde{\boldsymbol{\pi}} = \underbrace{\begin{pmatrix} U_d \\ U_s \end{pmatrix}}_{\text{PE}} + \underbrace{\begin{pmatrix} (\tilde{\lambda}_1 x + \tilde{\lambda}_2 z)U_d \\ (\tilde{\lambda}_1 z + \tilde{\lambda}_2 x)U_s \end{pmatrix}}_{\text{GE: own-shock}} + \underbrace{(\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu \begin{pmatrix} U_s \\ U_d \end{pmatrix}}_{\text{GE: confusion}},$$

where

$$\tilde{\lambda}_k = \frac{\delta\lambda_k}{1 - \delta\lambda_k}, \qquad k = 1, 2,$$

and λ_k *is an eigenvalue of* $\mathbf{P}$*.*

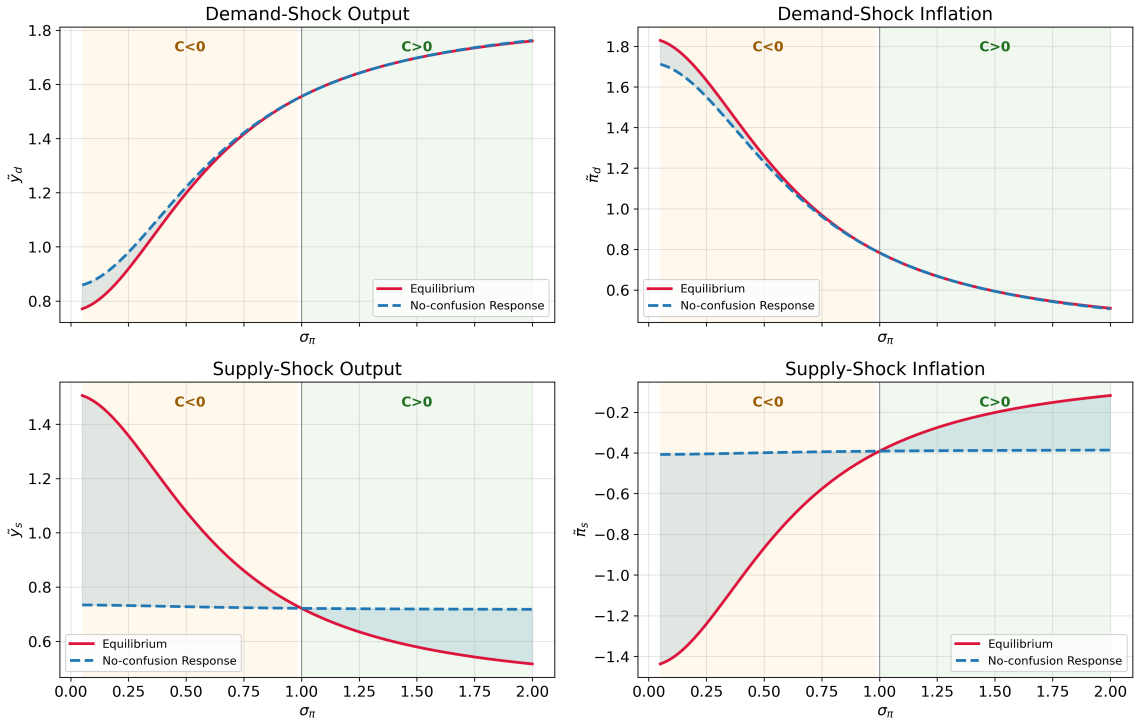
The theorem decomposes inflation responses into three components. The first is the partial-equilibrium response, **U**, which captures firms' direct pricing response given their beliefs. The second is own-shock GE feedback: the initial PE response triggers further rounds of strategic pricing responses by other firms. It corresponds to the special case in which **P** is diagonal. The third term is new. It is the GE effect of higher-order confusion. Because **P** is not diagonal, the demand-shock inflation response loads on U_s, while the supply-shock inflation response loads on U_d. The strength of this cross-shock feedback is governed by $(\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu$, which is essentially the sum of higher-order confusion weighted by the strategic complementarity δ.

This decomposition separates standard GE attenuation from confusion-driven GE feedback. In standard incomplete-information models, information frictions dampen GE responses because higher-order expectations place less weight on the true shock and remain more anchored to the prior (Angeletos and Lian, 2023). Confusion operates along a different dimension: it changes the *direction*, not only the *magnitude*, of GE feedback. The PE response to a shock affects the responses to the other shock through GE feedback because agents place weight on the wrong shock in their beliefs. The sign and strength of this cross-shock feedback are governed by confusion. Importantly, GE attenuation and GE confusion are distinct objects. A single precise signal paired with an uninformative one can generate moderate GE attenuation but maximal confusion. Conversely, two similarly precise signals can imply comparable GE attenuation but little confusion.

For illustration, we solve the general-equilibrium system numerically and plot the responses in Figure 3. We set shock variances equal to 0.25, output signal noise variance and signal loadings to be one. The top panels show responses to a unit demand shock as σ_π varies, while the bottom panels show responses to a unit supply shock. The red solid lines report the full equilibrium responses with confusion. The blue dashed lines report the no-confusion component, obtained by removing all terms involving C from both the PE and GE responses.

Consistent with Proposition 2, negative confusion is strongest as $\sigma_\pi \to 0$. The effect is especially pronounced after a supply shock. When the inflation signal is nearly precise, the full equilibrium responses of inflation and output are substantially larger than the

Figure 3: Effect of Confusion on Output and Inflation



Notes: The figure shows the responses of inflation and output to demand and supply shocks. The top panels report responses to a unit demand shock for different values of σ_π. The bottom panels report responses to a unit supply shock. The red solid lines show the full equilibrium responses with confusion. The blue dashed lines show the no-confusion component, obtained by removing all terms involving C from both the PE and GE responses. Also, $\sigma_d = \sigma_a = 0.5$, $\sigma_y = 1$, and signal loadings are set to be one. We use the remaining parameter values from Table 2

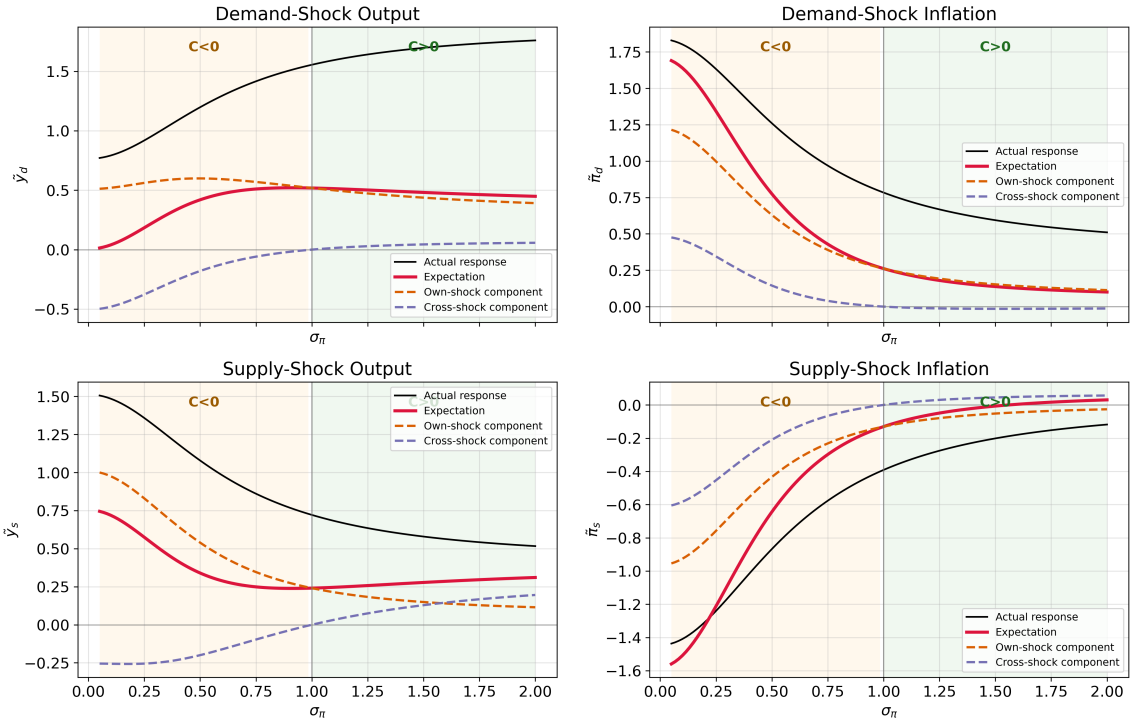
no-confusion component. Moreover, most of the variation in supply-shock responses across σ_π is driven by confusion. As σ_π rises, negative confusion weakens. With $\sigma_y = 1$ and unit loadings, confusion disappears at $\sigma_\pi = 1$ and reverses sign thereafter.

For demand shocks, confusion has a smaller effect. Inflation and output move modestly relative to the no-confusion component. The no-confusion component instead displays the standard information-attenuation mechanism. When signals are precise, firms infer more of the underlying demand shock, so prices adjust more and output responds less. As signals become noisier, firms underreact in prices, and the output response becomes larger. This is the monetary non-neutrality mechanism emphasized by Woodford (2003).

Expectations We next analyze expectation responses. Expected inflation after demand and supply shocks can be written as

$$\mathbb{E}\tilde{\pi}_d = D_d\tilde{\pi}_d + C\tilde{\pi}_s, \qquad \mathbb{E}\tilde{\pi}_s = D_s\tilde{\pi}_s + C\tilde{\pi}_d,$$

Figure 4: Effect of Confusion on Expected Output and Inflation



Notes: The figure decomposes expected inflation and output responses under exogenous information as σ_π varies. Black solid lines are actual equilibrium responses. Red solid lines are total expectations. Dashed lines decompose each expectation into the component coming from the own-shock response and the component coming from the cross-shock response.

with analogous expressions for expected output.

Figure 4 plots firms' expected responses of inflation and output, decomposed into own-shock components, such as $D_d\tilde{\pi}_d$, and cross-shock components, such as $C\tilde{\pi}_s$. Expected output after a demand shock is substantially below actual output, especially when the inflation signal is precise. This gap is driven mainly by confusion. When $C < 0$, firms partly interpret a positive demand shock as an adverse supply. Since adverse supply lowers output, the cross-shock component depresses expected output. This force is strongest as $\sigma_\pi \to 0$, when inflation is highly informative and negative confusion is largest.

The own-shock component reflects two opposing forces. On the one hand, more precise information allows firms to infer more of the true demand shock, which induces stronger price adjustment and reduces output response. On the other hand, more precise information also implies that firms believe larger movements in equilibrium output and inflation. The interaction of these forces makes expected demand-driven output non-monotone in σ_π.

For supply shocks, more precise information raises own-shock perception: firms perceive a larger supply shock, cut prices more, and expect output to rise more. The cross-shock component works in the opposite direction when σ_π is small. Negative confusion makes firms infer weak demand, which lowers expected output. Combining these two forces, expected output after a supply shock can first fall and then rise with σ_π.

In sum, a precise inflation signal makes firms perceive demand-driven output movements as small relative to supply-driven movements. Section 4 shows that this mechanism has a direct empirical implication: survey forecasts of output are primarily driven by supply shocks, even though realized output may be driven more by demand shocks.

3.2 Endogenous Information

So far, we have treated the loadings in the output and inflation signals as exogenous. We now endogenize information by tying these loadings to the general-equilibrium responses of output and inflation. We first establish existence and uniqueness, and then study how confusion changes when information is endogenous.

Endogenous information generates strong *information complementarity*. One firm's price choice becomes part of the aggregate signal observed by other firms. This complementarity can produce multiple equilibria. In an economy with only a demand shock and an inflation signal, Proposition A.1 shows that a zero-inflation equilibrium always exists. If all firms keep prices unchanged, inflation contains no information about the demand shock. Firms therefore fail to infer the shock, and price inaction becomes self-confirming.

Proposition A.3 shows that this logic survives with the setup where both demand and supply shocks can occur simultaneously. Suppose the inflation signal is informative but the output signal is uninformative. Then a zero-inflation response to a demand shock can be sustained: if inflation does not move after the demand shock, the inflation signal carries no demand-shock information, and any inflation movement is attributed to supply. In this equilibrium, demand shocks generate no inflation response, supply shocks generate nonzero inflation responses, and confusion vanishes along the demand-shock dimension. A second, nonzero-inflation equilibrium may also arise when firms coordinate on price adjustment, thereby making inflation informative about demand. If the inflation signal

is sufficiently noisy, this nonzero equilibrium disappears because price deviations are too small to be detected.

Adding an output signal can restore uniqueness. Proposition A.2 shows that a sufficiently informative output signal eliminates the zero-inflation equilibrium. Intuitively, if firms initially refrain from changing prices after a demand shock, output still rises. Because firms also observe an output signal, they infer the presence of a demand shock and raise prices. Naturally, this result holds only when output signal is informative enough.

Endogenous information changes confusion by making agents learn more from the shocks that move observable aggregates more strongly. If inflation is primarily supply-driven, then inflation becomes a better signal of supply shocks. As a result, agents are less likely to misattribute a true supply shock to demand, but more likely to misattribute a true demand shock to supply. Output, by contrast, is typically driven more evenly by demand and supply, so output-based learning can generate sizable confusion in both directions. Figure A.1 illustrates how endogenous information reshapes responses relative to the exogenous-information case.

Summary. Taken together, the simple New Keynesian model delivers four lessons. First, confusion can substantially reshape equilibrium responses, especially to supply shocks. Second, higher-order confusion enters through GE feedback: whereas standard GE attenuation dampens responses, confusion redirects feedback across shocks and can operate in the opposite direction. Third, confusion distorts expectations. When the inflation signal is precise, firms' expected output response to demand shocks can be muted or even negative. Fourth, endogenous information can generate multiplicity and changes the inference: agents learn most from shocks that leave the largest footprint on observable aggregates.

4 Quantitative Model

In this section, we develop a dynamic quantitative model of confusion with endogenous information. The model follows closely the logic of an island economy as in Lucas (1972) and Lorenzoni (2009).

Households. The economy is populated by a unit mass of households indexed by $i \in [0,1]$. Household i chooses consumption, labor supply, and nominal bond holdings to maximize

$$\max_{\{C_{it+k}, N_{it+k}, B_{it+k+1}\}_{k\geq 0}} \mathbb{E}_{it} \sum_{k=0}^{\infty} \left(\prod_{s=0}^{k-1} \tilde{\beta}_{i,t+s} \right) \left[\frac{(C_{it+k} - \lambda_c C_{i,t+k-1})^{1-\sigma}}{1-\sigma} - \frac{N_{it+k}^{1+\eta}}{1+\eta} \right], \tag{9}$$

where $\lambda_c \in [0,1)$ governs external habit formation, $\sigma > 0$ is the coefficient of relative risk aversion, and $\eta > 0$ is the inverse Frisch elasticity. The household discount-factor satisfies

$$\log \tilde{\beta}_{it} = \log \beta + \beta_t + \xi_{it}, \qquad \beta_t = \rho_d \beta_{t-1} + u_t^d, \tag{10}$$

where β is the steady-state discount factor, β_t is the aggregate demand shifter, and $\xi_{it} \sim \mathcal{N}(0, \sigma_{di}^2)$ is an idiosyncratic preference component independent of $u_t^d \sim \mathcal{N}(0, \sigma_d^2)$. Thus, a household's own discount factor is informative about aggregate demand. The budget constraint is

$$P_{it} C_{it} + B_{it+1} = R_t B_{it} + W_{it} N_{it} + \Pi_{it}, \tag{11}$$

with bonds in zero net supply. Here P_{it} is the household-specific price index, R_t is the gross nominal interest rate, W_{it} is the nominal wage, and Π_{it} denotes rebated profits.

As in island models, households do not observe the aggregate price level perfectly. In particular, each household purchases a non-representative basket of differentiated goods,

$$C_{it} = \left(\int_{\Omega_{it}} C_{ijt}^{\frac{\epsilon-1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon-1}}, \qquad P_{it} = \left(\int_{\Omega_{it}} P_{jt}^{1-\epsilon} \, dj \right)^{\frac{1}{1-\epsilon}}, \tag{12}$$

where $\epsilon > 1$ and Ω_{it} is household i's consumption basket. We interpret P_{it} as a local signal of the aggregate price level P_t: it contains information about aggregate inflation but does not reveal it perfectly (Lorenzoni, 2009).

Labor Unions and Segmented Labor Markets. To prevent wages from fully revealing aggregate conditions, we introduce segmented labor markets indexed by $k \in [0,1]$. Households supply differentiated labor services to labor unions. Each union k aggregates labor from a random subset of households, Ω_k, into a union-specific labor bundle, together with

associated nominal wage index,

$$N_{kt} = \left(\int_{\Omega_k} N_{ikt}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \qquad W_{kt} = \left(\int_{\Omega_k} W_{ikt}^{1-\epsilon_w} \, di \right)^{\frac{1}{1-\epsilon_w}}. \tag{13}$$

where $\epsilon_w > 1$ is the elasticity of substitution across labor types. Each firm hires labor from only one union. As a result, a firm observes the wage index W_{kt} of its own labor supplier rather than the aggregate nominal wage. Because each union aggregates a random subset of households, W_{kt} is informative about aggregate labor-market conditions but does not perfectly reveal the aggregate state.

Each union sets nominal wages for its member households, taking the labor-demand schedule for its bundle as given. The union sets the real wage as a constant markup over the household marginal rate of substitution:

$$\frac{W_{ikt}}{P_{it}} = \mu_w (C_{it} - \lambda_c C_{it-1})^{\sigma} N_{ikt}^{\eta}, \qquad \mu_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}. \tag{14}$$

Firms. A unit mass of monopolistically competitive firms, indexed by $j \in [0, 1]$, produces differentiated goods using labor hired from a single labor union. Each firm sells to a local set of consumers with market-clearing condition $Y_{jt} = \int C_{ijt} di$. Production is $Y_{jt} = A_{jt} N_{jt}^{1-\alpha}$, where A_{jt} is firm-specific productivity. Firm-level log productivity is

$$a_{jt} = a_t + \xi_{jt}, \qquad a_t = \rho_a a_{t-1} + u_t^a,$$

where $u_t^a \sim \mathcal{N}(0, \sigma_a^2)$ is the aggregate productivity innovation and $\xi_{jt} \sim \mathcal{N}(0, \sigma_{ai}^2)$ is the idiosyncratic productivity innovation. Thus, each firm's own productivity realization a_{jt} is a private signal about a_t.

Price setting is subject to Calvo rigidity. In each period, a firm can reset its price with probability $1 - \theta$ and otherwise keeps its previous price unchanged. A resetting firm chooses P_{jt}^* to maximize

$$\max_P \; \mathbb{E}_{jt} \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \Big[P \, Y_{j,t+k}(P) - W_{j,t+k} N_{j,t+k}(P) \Big], \tag{15}$$

where $Q_{t,t+k}$ is the stochastic discount factor and $W_{j,t+k}$ is the wage charged by the labor union from which firm j hires. When resetting prices, firms forecast aggregate demand, price level, and real wage using their information at time t. After prices are posted, demand is realized; each firm hires from its matched labor union and produces to meet demand.

Monetary Authority. The central bank sets the nominal interest rate according to a standard Taylor rule:

$$R_t = \bar{R}\Pi_t^{\phi_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}, \tag{16}$$

where $\bar{R}$ and $\bar{Y}$ denote steady-state values and Π_t is gross inflation. We assume the central bank observes the aggregate state perfectly.

Information Structure. The economy is driven by aggregate demand and supply shocks. Neither shock is directly observed. Agents instead observe local outcomes that are informative but only partially revealing. Firms observe their own productivity, local demand, and the wage charged by the labor union from which they hire. Households observe their own discount-factor component, local price index, and interest rate.

For tractability, we summarize this rich information environment with two endogenous signals: one about aggregate output and one about aggregate inflation. This simplification is without loss for our purposes because the local outcomes that agents observe in equilibrium are linear combinations of output and inflation. In particular, we assume that households observe their own demand shifter b_{it} and the policy rate i_t, while firms observe their own productivity a_{jt}. All agents observe noisy signals of output and inflation:

$$x^y_{\ell t} = y_t + \xi^y_{\ell t}, \qquad x^\pi_{\ell t} = \pi_t + \xi^\pi_{\ell t}, \qquad \ell \in \{i, j\}.$$

Thus the information sets are

$$\mathcal{I}_{it} = \{b_{it},\, x^y_{it},\, x^\pi_{it},\, i_t,\, \mathcal{I}_{i,t-1}\}, \qquad \mathcal{I}_{jt} = \{a_{jt},\, x^y_{jt},\, x^\pi_{jt},\, \mathcal{I}_{j,t-1}\}.$$

Equilibrium. Equations (9)–(16), together with market-clearing conditions, characterize the equilibrium.

Log-linearized Equilibrium Conditions. We log-linearize the model around the steady state. Under dispersed information, the representative-agent recursive formulation is not available. Nevertheless, the equilibrium can be written recursively at the individual level.

With external habit, the household Euler equation becomes

$$c_{it} = \frac{\lambda_c}{1+\lambda_c} c_{i,t-1} + \frac{1}{1+\lambda_c} \mathbb{E}_{it} c_{i,t+1} - \frac{1-\lambda_c}{(1+\lambda_c)\sigma} \left(i_t - \mathbb{E}_{it} \pi_{t+1} - b_{it} \right). \tag{17}$$

Households observe the nominal policy rate i_t exactly, but differ in perceived ex ante real rates because they form different inflation expectations. Habit dampens initial output responses, especially to demand shocks, and helps generate plausible confusion. Since agents infer from the aggregates the shock leaves the largest footprint, an excessively large demand-driven output response would tilt inference toward positive confusion.

Define the reset-price gap as $z_{jt} \equiv p^*_{jt} - p_{t-1}$. The aggregate inflation thus satisfies $\pi_t = (1-\theta) \int_0^1 z_{jt}\, dj$. Combining firms' optimal pricing condition with the habit-adjusted wage equation derived in Appendix B.1, the log-linear reset-price equation is then[4]

$$z_{jt} = \mathbb{E}_{jt} \pi_t + (1-\beta\theta) \left\{ \psi'_y \mathbb{E}_{jt} y_t - \psi_\lambda \mathbb{E}_{jt} y_{t-1} - \eta \psi_a \mathbb{E}_{jt} a_t - \psi_a a_{jt} \right\} + \beta\theta \mathbb{E}_{jt} z_{j,t+1}, \tag{18}$$

where $\psi'_y \equiv \frac{\varphi}{\varepsilon} + (1-\varphi)\left(\frac{\sigma}{1-\lambda_c} + \frac{\eta}{1-\alpha} \right)$, $\psi_\lambda \equiv (1-\varphi)\frac{\sigma\lambda_c}{1-\lambda_c}$, $\psi_a \equiv \frac{1-\varphi}{1-\alpha}$.

Finally, the log-linearized monetary rule is

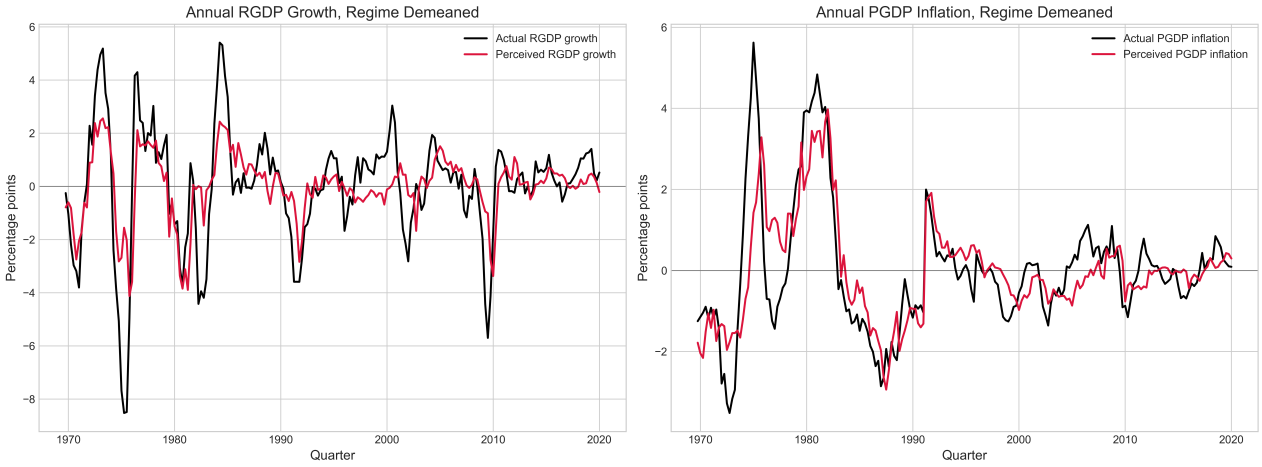
$$i_t = \phi_\pi \pi_t + \phi_y y_t. \tag{19}$$

5 Estimation

We estimate the model separately before and after 1990. The estimates discipline both shock variances and signal informativeness, which determine the sign and magnitude of confusion. We then use the estimated model to study historical decompositions, impulse responses, and policy counterfactuals.

[4] Following Angeletos and Lian (2018) and Angeletos and Huo (2021), we assume that firms know p_{t-1} but do not extract additional information from it. This assumption simplifies aggregation and keeps the focus on the information contained in current endogenous outcomes.

Figure 5: Realized and Expected Real-Time RGDP Growth and PGDP Inflation



Notes: The left panel plots annual actual and perceived GDP growth; the right panel plots annual actual and perceived GDP-deflator inflation. Solid black lines denote real-time realized growth rates. Colored lines denote SPF mean expectations for the corresponding annual growth rate, constructed from level forecasts and aligned with the terminal quarter of the forecast horizon. All series are demeaned by their own mean within monetary-policy regimes. Units are percentage points.

Data. Our empirical moments are constructed from real-time U.S. macroeconomic data and survey expectations. Realized inflation and output are computed from Philadelphia Fed real-time data on the GDP price deflator and real GDP, while perceived PGDP inflation and RGDP growth are computed from the Survey of Professional Forecasters. We use annual growth rates and restrict the sample to 1969Q3–2019Q4, excluding the pandemic period because it represents a distinct regime relative to the Great Moderation and the post-Great-Recession period. In Section 5.1, we study a counterfactual designed to mimic key features of this period and draw policy implications.

For realized growth, we impose a real-time information convention. The observation for quarter t is computed using the vintage released in quarter $t+1$, and both the current and lagged levels are taken from that same vintage. This same-vintage construction ensures that measured growth reflects information available in real time rather than subsequent revisions. For perceived growth, we use the SPF four-quarter-ahead level forecasts to form the survey-implied annual growth rate over the same four-quarter interval as the realized series. Thus, the perceived object is the forecast of the annual growth rate realized over that window.

We remove low-frequency shifts by demeaning each series within broad commonly recognized regimes. The regimes are pre-Volcker through 1979Q2, Volcker disinflation from 1979Q3 through 1990Q4, Great Moderation from 1991Q1 through 2007Q4, and post-2008 from 2008Q1 through 2019Q4. We apply this transformation separately to each variable. In particular, perceived variables are demeaned using their own regime means, rather than the realized means, so that average expectation wedges within a regime are not mechanically removed. We use regime demeaning rather than smooth filters because low-frequency movements in inflation and output may reflect trends agents learned in real time. Filtering them out would mechanically weaken the comovement between realized and perceived variables and bias upward the estimated information friction, especially in the post-Volcker period (Erceg and Levin, 2003). Regime demeaning removes only broad regime-level mean shifts, while preserving within-regime variation.

The data show that expectations move substantially over time. Professional forecasters track realized inflation and real-time GDP growth closely, but they miss the sharp rise in inflation and the contraction in GDP growth in the early 1970s, when the economy was hit by large and persistent oil shocks. During the post-Volcker disinflation from 1980 to 1990, forecasts continue to track inflation closely, but follow GDP growth less tightly. This is a period in which output became especially difficult to forecast: oil prices fell a lot, while monetary policy remained tight. After 1990, inflation expectations become more anchored, and both macroeconomic volatility and forecast-realization comovement decline. This pattern is consistent with recent evidence that lower macroeconomic volatility reduces incentives to acquire and process information (Weber et al., 2023). We therefore split the sample into pre-1990 and post-1990 periods, following Hazell et al. (2022).

Estimation Method. We estimate the model separately on the pre-1990 and post-1990 samples. The parameter vector is $\Theta = (\sigma_d^2, \sigma_a^2, \sigma_\pi^2, \sigma_y^2)$, where σ_d^2 and σ_a^2 are the variances of demand and supply shocks, and σ_π^2 and σ_y^2 are the variances of inflation and output signal noise. All other structural parameters are calibrated.

The empirical targets are the ten unique elements of the contemporaneous covariance matrix of annual regime-demeaned data. Let $x_t = (y_t, \pi_t, \mathbb{E}_t y_t, \mathbb{E}_t \pi_t)'$. We target

$$m^{data} = \text{vech}\left(\Sigma^{data}\right), \qquad \Sigma^{data} = \frac{1}{T}\sum_t (x_t - \bar{x})(x_t - \bar{x})'.$$

All variables are annual four-quarter objects. We focus on firms' expectations, since the quantitative exercise studies how firms use inflation and output signals when setting prices. Household expectations may also reflect the central bank's signaling role, as emphasized by Melosi (2017), which is not the focus here.

For each Θ, we solve the model and compute impulse responses to demand and supply shocks. Since the empirical variables are annual growth rates, the model moments are computed from annual responses. For shock $s \in \{d, a\}$, define

$$b_h^s = \left(\sum_{\ell=0}^{3} y_{h+\ell}^s, \sum_{\ell=0}^{3} \pi_{h+\ell}^s, \sum_{\ell=0}^{3} \mathbb{E} y_{h+\ell}^s, \sum_{\ell=0}^{3} \mathbb{E}\pi_{h+\ell}^s\right)'.$$

The model-implied covariance matrix is

$$\Sigma^{model}(\Theta) = \sigma_d^2 \sum_{h=0}^{H} b_h^d b_h^{d\prime} + \sigma_a^2 \sum_{h=0}^{H} b_h^a b_h^{a\prime}.$$

We estimate Θ by minimizing weighted absolute relative deviations:

$$\widehat{\Theta} = \arg\min_{\Theta} \sum_k \left| \frac{m_k^{model}(\Theta) - m_k^{data}}{m_k^{data}} \right|.$$

Identification The moments discipline these parameters in distinct ways. Realized variances and the covariance between output and inflation discipline the variances of demand and supply shocks. The variances of expectations discipline the overall precision of information. The covariances between realized and expected variables discipline the relative informativeness of output and inflation signals, and therefore the sign and magnitude of confusion. In particular, stronger comovement between inflation and expected inflation pushes the model toward a more precise inflation signal, while stronger comovement between output and expected output pushes it toward a more precise output signal.

Computational Challenges. Solving a dynamic beauty-contest model with endogenous information is computationally demanding. We solve the model using the analytic policy-function iteration approach of Han et al. (2022), which evaluates the annihilation operator via the discrete Fourier transformation, and we construct expectations using the Wiener-Hopf filter as in Huo and Takayama (2025). Building on this computational machinery, we introduce numerical refinements that reduce redundant computation and improve the stability of the fixed-point iteration. In our implementation, these refinements reduce the solution time of a standard DSGE model with information frictions and endogenous information by roughly one half. Details are provided in Appendix B.

Estimation Results. Table 1 shows that the model fits the targeted contemporaneous moments reasonably well in both samples. It captures the covariance structure of realized variables and, more importantly, the covariance between realized and expected variables. This covariance is central because its relative strength across output and inflation disciplines the sign and magnitude of confusion. In the pre-1990 sample, the model also matches the variances of expectations well, although it overstates some expectation covariances. In the post-1990 sample, it fits expectation covariances well but understates their variances. Given the model's two-shock structure and the likely measurement error in expectation data, these discrepancies are not surprising.

The calibrated parameters in Table 2 are standard in the New Keynesian literature. We set $\beta = 0.99$, $\epsilon = 6$, $\alpha = 0.33$, and Taylor-rule coefficients $\phi_\pi = 1.5$ and $\phi_y = 0.5$. Shock persistence is fixed at 0.8. We do not estimate persistence because doing so is computationally costly and would blur the role of information frictions: as Section 2.3 shows, asymmetric persistence itself affects learning and confusion. To assess whether the fixed persistence is reasonable, Tables B.2 and B.3 report untargeted autocovariance matrices through lag 3. Although these moments are not targeted, the model captures the broad persistence patterns in the data.

Habit formation is set to $\lambda_c = 0.6$, which generates a moderate on-impact output response to demand shocks. We set $\theta = 0.5$ so that the estimation focuses on information

Table 1: Model vs. Data Moments: Contemporaneous Covariance Matrices

Pre-1990

	Data				*Model*			
	y_t	π_t	$E_t y_t$	$E_t \pi_t$	y_t	π_t	$E_t y_t$	$E_t \pi_t$
y_t	8.93	-4.38	3.86	-2.12	8.93	-3.46	4.24	-3.07
π_t	-4.38	5.57	-2.61	3.51	-3.46	5.11	-3.07	4.00
$E_t y_t$	3.86	-2.61	2.90	-1.54	4.24	-3.07	2.90	-2.70
$E_t \pi_t$	-2.12	3.51	-1.54	3.39	-3.07	4.00	-2.70	3.39

Post-1990

	Data				*Model*			
	y_t	π_t	$E_t y_t$	$E_t \pi_t$	y_t	π_t	$E_t y_t$	$E_t \pi_t$
y_t	1.93	-0.24	0.54	-0.39	1.93	-0.24	0.69	-0.15
π_t	-0.24	0.37	0.06	0.17	-0.24	0.37	-0.15	0.17
$E_t y_t$	0.54	0.06	0.56	-0.14	0.69	-0.15	0.34	-0.10
$E_t \pi_t$	-0.39	0.17	-0.14	0.27	-0.15	0.17	-0.10	0.10

Table 2: Calibrated and Estimated Parameters

Calibrated parameters

Parameter	Description	Value	Parameter	Description	Value
β	Discount factor	0.99	ϕ_y	Taylor rule response to output	0.5
σ	Inverse intertemporal elasticity	1	λ_c	External habit	0.6
κ	Inverse elasticity of labor supply	1	σ_{ai}/σ_a	Exogenous supply-signal noise	20
α	Decreasing returns to scale	0.33	σ_{di}/σ_d	Exogenous demand-signal noise	20
ϵ	Elasticity of substitution across goods	6	ρ_d	Persistence of demand shock	0.8
θ	Calvo price stickiness	0.5	ρ_a	Persistence of supply shock	0.8
ϕ_π	Taylor rule response to inflation	1.5			

Estimation: 1969Q3–1989Q4			**Estimation: 1991Q1–2019Q4**		
Parameter	**Description**	**Value**	**Parameter**	**Description**	**Value**
σ_d^2	Demand-shock variance	0.1106	σ_d^2	Demand-shock variance	0.0314
σ_a^2	Supply-shock variance	0.5354	σ_a^2	Supply-shock variance	0.1433
σ_π^2	Inflation signal-noise variance	0.1288	σ_π^2	Inflation signal-noise variance	0.1012
σ_y^2	Output signal-noise variance	3.4480	σ_y^2	Output signal-noise variance	0.7608

frictions rather than strong nominal rigidity.[5] Exogenous signal standard deviations are

[5]Although Nakamura and Steinsson (2008) find that prices readjust every three to four quarters, some measured price stickiness does not translate into monetary non-neutrality because of selection effects, as emphasized by Golosov and Lucas (2007). Auclert et al. (2024) show that, after accounting for selection, the empirical degree of price stickiness is equivalent to a Calvo parameter of about 0.5, or an average price duration of two quarters.

fixed at twenty times the corresponding aggregate shock standard deviation, so agents do not observe informative private signals. These exogenous signals help deliver equilibrium uniqueness and numerical stability.

The estimates in Table 2 imply much larger demand- and supply-shock variances before 1990. This accords with the historical view that monetary and oil supply shocks were central sources of fluctuations in this episode. After 1990, both variances fall sharply, consistent with the decline in aggregate volatility during the Great Moderation. The correlation between inflation and output becomes less negative, indicating demand shocks account for more of the variations in the post-1990 sample.

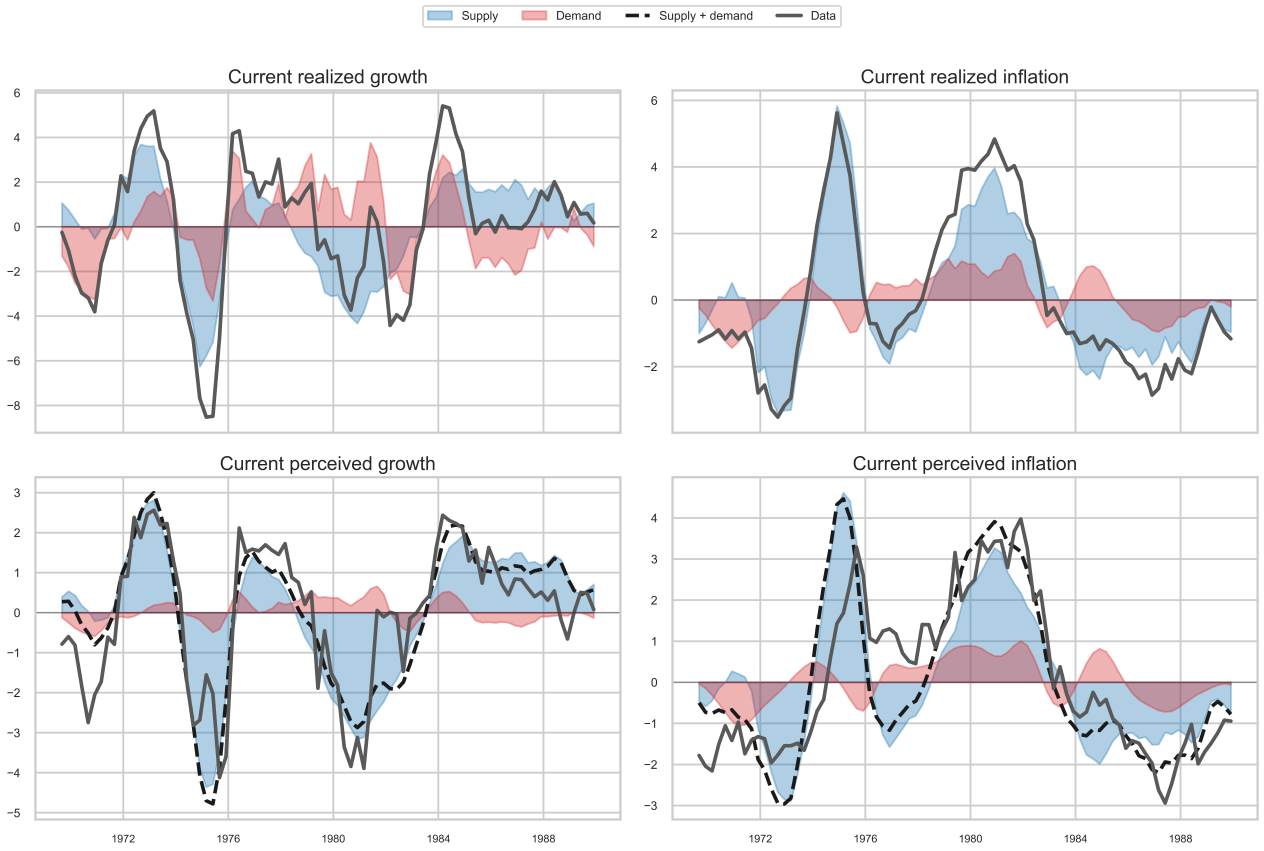
In the pre-1990 sample, the high covariance between inflation and expected inflation pushes the estimation toward a highly precise inflation signal. By contrast, the estimated output signal is much noisier, despite the moderate covariance between output and expected output. The model reconciles these facts through negative confusion. The asymmetry in signal precision leads agents to attribute much of perceived output variation to supply shocks rather than demand shocks. Perceived supply shocks move expected output substantially, generating both moderate expected-output volatility and a positive covariance between output and expected output. We show this mechanism directly below using the model-implied decomposition of expectations.

In the post-1990 sample, the covariance between inflation and expected inflation is much smaller, pushing the estimation toward a much noisier inflation signal. At the same time, the output signal becomes relatively more precise. This helps explain the post-1990 moment fit. Since demand shocks move output more strongly than inflation, a more precise output signal provides sharper information about demand shocks. Demand shocks therefore account for more of the variation in expected output, raising the covariance between realized and expected output relative to the pre-1990 sample.

5.1 Results

This section decomposes historical fluctuations into demand and supply shocks and evaluates the model's fit to expectation dynamics. We show that confusion provides a plausible explanation for the "supply-side view" in the literature. We show substantially

Figure 6: Historical Decomposition Before 1990



Notes: Stacked-area plots attribute quarterly fluctuations in real GDP, inflation, and their SPF expectations, before 1990, to the structural demand (blue) and supply (red) shocks recovered via model inversion. The first row shows the results for real GDP and its forecast, while the second row shows the results for inflation and its forecast.

stronger pre-1990 responses to supply shocks. Then, we argue that the large output response to demand shocks and high covariance between output and expected output can co-exist, and document the post-1990 flattening of the Phillips curve. Finally, we study a counterfactual with more accommodative monetary policy and draw policy implications.

Historical Decomposition and the Supply-side View. We use the observed joint time series of real GDP growth and inflation to recover the sequence of structural shocks by model inversion. Since the model has two structural shocks and two observables, the inverted shocks match the realized series exactly by construction. We then feed these shocks back through the model to generate expectations. The expectation series are not used in the inversion and therefore provide an out-of-sample check on the estimation.

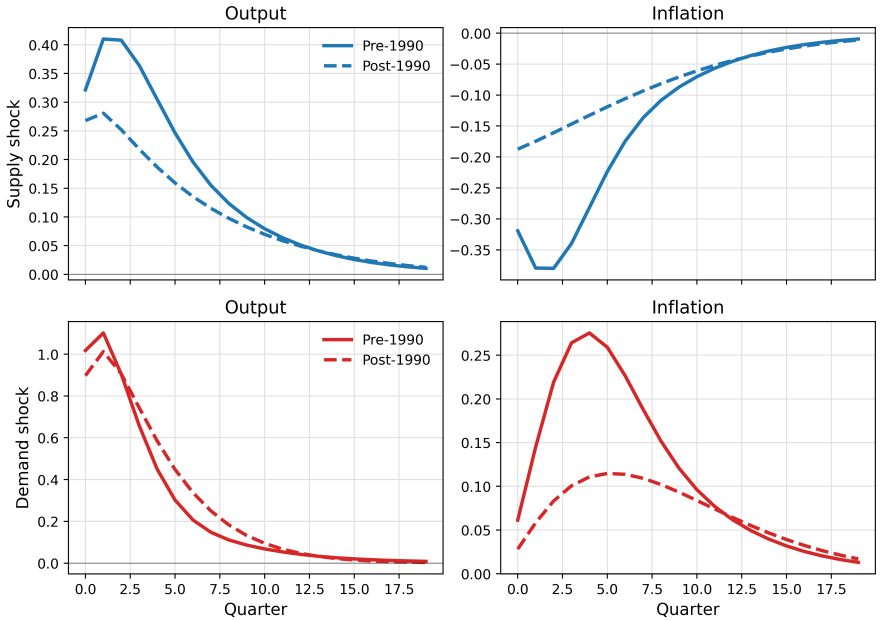
Figures 6 compares SPF expectations, shown in solid lines, with model-implied expectations, shown in dashed lines. The model fits the expectation dynamics. Furthermore, it decomposes pre-1990 output, inflation, and expectations into demand and supply components. Realized output fluctuations are more supply-driven, with demand accounting for 38% of output variance and supply for 62%. Expected output, however, is almost entirely supply driven: supply shocks account for 97% of its variance. Realized and expected inflation are also mostly supply driven, with supply shocks accounting for roughly 90% of their variances.

This pattern reflects the model's core confusion mechanism. Inflation is observed relatively precisely, while output is noisy. This has two implications. First, because demand shocks account for much of the variation in output, a noisy output signal makes demand shocks hard to identify. Second, the imbalance in signal precision generates negative confusion. Agents therefore attribute much of perceived output variation to supply shocks, and these perceived supply shocks move expected output. Perceived inflation is also largely supply driven. Together, these forces generate a *supply-side view* of aggregate fluctuations, consistent with recent survey evidence (Weber et al., 2022; Candia et al., 2023a).

Figure B.1 reports the post-1990 fit and decomposition. Realized output fluctuations become predominantly demand driven, with demand shocks accounting for about 71% of output variance. This marks a sharp shift from the pre-1990 decomposition and is consistent with the weaker negative comovement between output and inflation, as demand shocks dominate. Expected output also becomes less supply driven: demand shocks explain about 49% of its variance, compared with only about 3% before 1990. As the output signal becomes more informative, the supply-side view of output fluctuations weakens, although expectations still remain slightly tilted toward supply shocks. Inflation, however, remains primarily supply driven, with supply shocks accounting for about 90% of inflation variance.

Impulse Responses. Figure 7 displays the responses of output and inflation to one-percent demand and supply shocks in the models estimated before and after 1990. Several patterns stand out. Demand shocks generate substantial monetary non-neutrality in both samples. The peak output response is slightly larger before 1990, 1.10, compared with 1.01 after 1990, while the peak inflation response is much larger, 0.28, compared with 0.11. The difference is

Figure 7: Impulse Responses from the Estimated Model

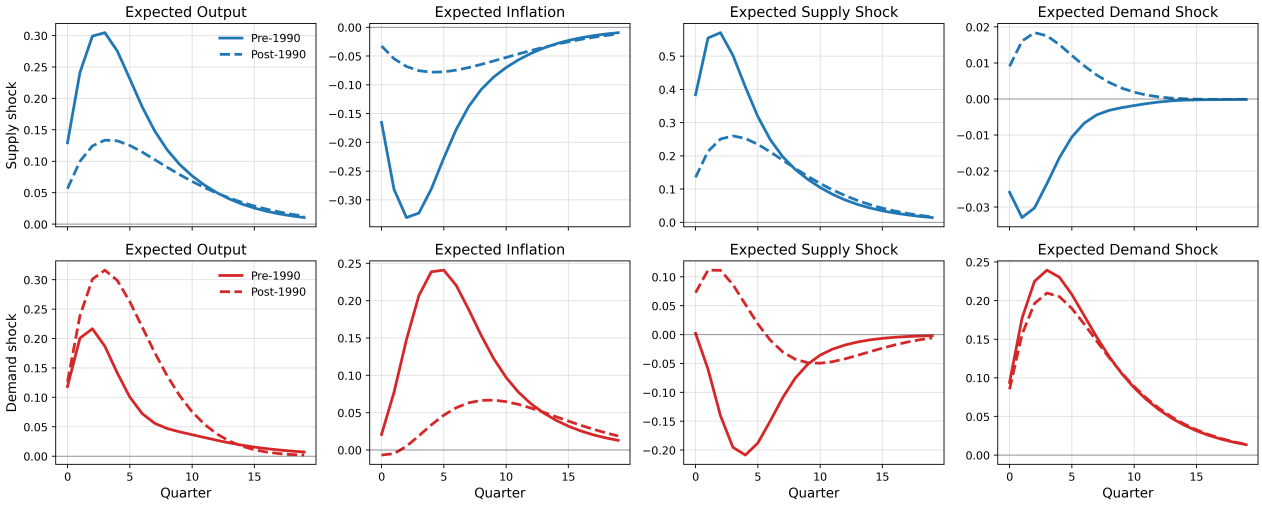


Notes: The figure plots current-quarter impulse responses of output and inflation using the pre- and post-1990 estimates in Table 2. Solid lines use the pre-1990 estimates; dashed lines use the post-1990 estimates. The first row reports responses to a supply-shock innovation, and the second row reports responses to a demand-shock innovation. Innovations are normalized to one percent, and horizons are quarterly.

also pronounced for supply shocks. The pre-1990 peak output response is 0.41, compared with 0.28 after 1990. The pre-1990 peak inflation response is -0.38, compared with -0.19 after 1990, roughly twice as large. Thus, the post-1990 Great Moderation reflects not only smaller shock variances, but also a substantial flattening of the Phillips curve and weaker transmission of supply shocks.

The figure shows large demand-driven output responses alongside the substantial movements in expectations documented in Figure 5. This combination is difficult to obtain in canonical single-shock incomplete-information models, as in Woodford (2003). Limited information about demand shocks is needed to generate the large output response and muted inflation response found in empirical impulse responses. Yet the same limited information would make expectations move too little. Our two-shock model breaks this tradeoff: expectations can move substantially even when demand shocks are poorly observed because agents misattribute movements between demand and supply shocks. The mechanism also fits the covariance between realized and expected output, strengthening its empirical plausibility.

Figure 8: Impulse Responses of Expectations



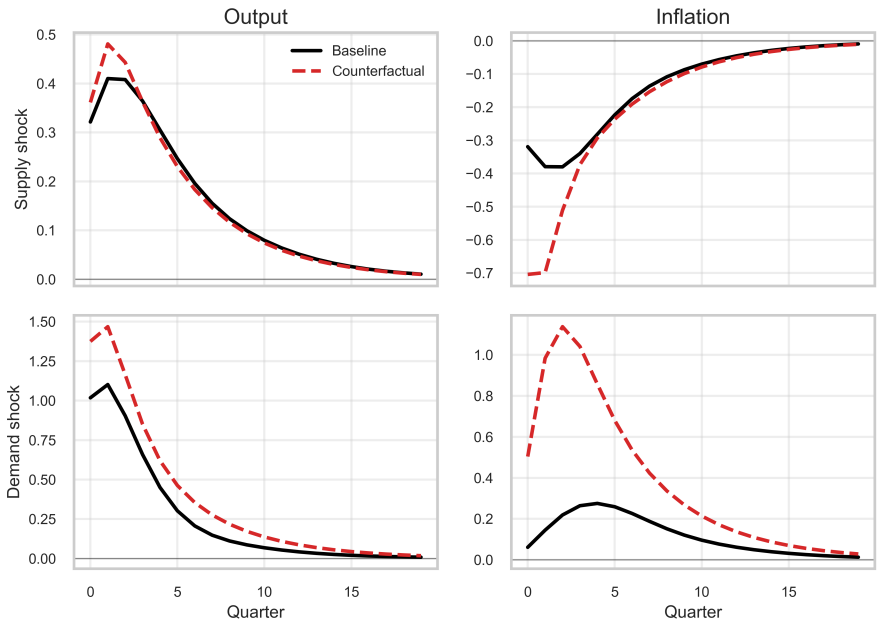
Notes: The figure plots current-period impulse responses of expectations using the pre- and post-1990 estimates in Table 2. The columns show expected output, expected inflation, the perceived supply shock, and the perceived demand shock. Innovations are normalized to one percent, and horizons are quarterly.

To understand why these patterns emerge, Figure 8 reports the belief dynamics. Solid lines show pre-1990 responses and dashed lines show post-1990 responses. Confusion is negative before 1990 and positive after 1990. After a true supply shock, the expected demand shock moves only modestly in both periods. Yet this small confusion matters because demand shocks generate large output responses and sizable inflation responses. This is visible in expected output and expected inflation: although the expected supply shock falls by only about one half after 1990, the peak decline in expected inflation becomes roughly four times smaller, and the peak response of expected output falls by more than one half. Thus, confusion accounts for a substantial part of expectation movements, which then generates larger responses of realized inflation and output before 1990.

After a demand shock, the expected supply shock falls sharply before 1990, reaching about -0.21, close in absolute value to own-shock perception, which peaks around 0.24. This negative perceived supply shock raises expected inflation substantially. After 1990, the sign reverses: the expected supply shock rises and peaks around 0.11, pushing expected inflation slightly below zero on impact.[6] The realized inflation response therefore falls from 0.28 before 1990 to 0.11 after 1990, a decline of about 60%, while the output response remains

[6] As the output response to the demand shock decays, confusion turns negative again.

Figure 9: Impulse Responses: Taylor-Rule Counterfactual



Notes: The figure plots current-quarter responses of output and inflation. Solid lines denote the baseline; dashed lines denote the counterfactual Taylor rule with $\phi_\pi = 1.1$ and $\phi_y = 0.2$. Shocks are one-percent innovations, and horizons are quarterly.

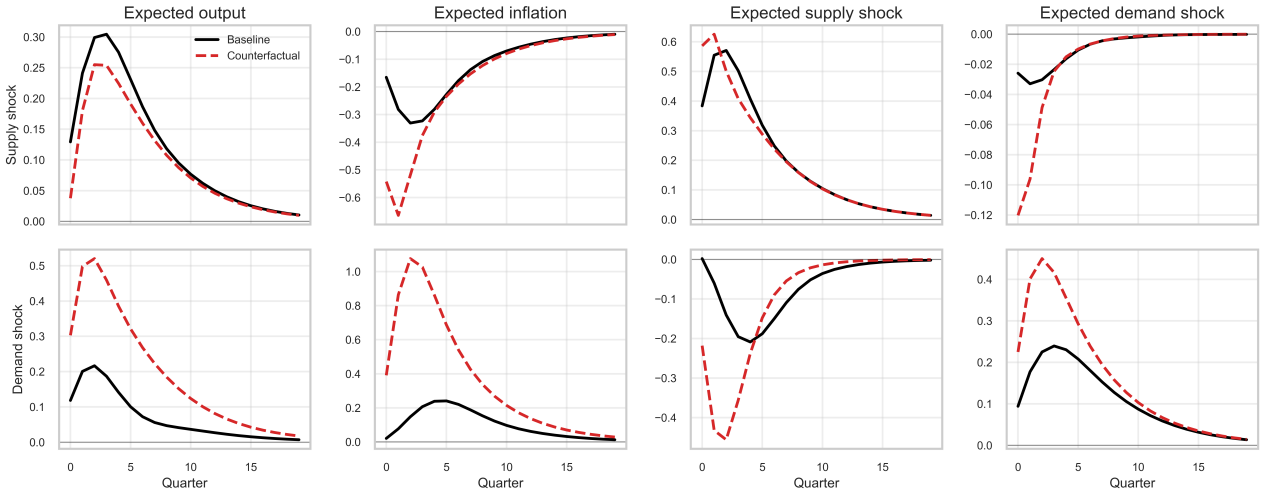
similar. Thus, firms' tendency to confuse demand shocks with adverse supply shocks is the fundamental reason why the Phillips curve is steeper before 1990 in the model. The implied post-1990 decline in the slope is consistent with Hazell et al. (2022).

Overall, confusion generates large expectation movements while preserving the large output and muted inflation responses to demand shocks, and provides a plausible explanation for the flattening of the Phillips curve. The same mechanism accounts for the supply-side view in survey evidence and explains why inflation responds to supply shocks more strongly when agents pay more attention to inflation.

Accommodative Monetary Policy. Figures 9 and 10 report the counterfactual exercise in which we keep the pre-1990 estimated shock and signal variances fixed and replace the baseline Taylor rule with a more accommodative rule, $\phi_\pi = 1.1$ and $\phi_y = 0.2$. This exercise is motivated by the pandemic period, in which monetary policy remained highly accommodative while households paid unusually close attention to inflation (Liu, 2025).

The effect is large. Under the counterfactual rule, the peak inflation response to a demand shock rises from 0.28 to 1.14. For a supply shock, the absolute inflation response rises from 0.38 to 0.70, almost doubling. Output responses also increase, especially for demand shocks,

Figure 10: Expectation Responses: Taylor-Rule Counterfactual



Notes: The figure plots current-period responses of expected output, expected inflation, the perceived supply shock, and the perceived demand shock. Solid lines denote the baseline; dashed lines denote the counterfactual Taylor rule with $\phi_\pi = 1.1$ and $\phi_y = 0.2$. Shocks are one-percent innovations, and horizons are quarterly.

with the peak output response rising from 1.10 to 1.47. Thus, when policy is more lenient, the same one-percent shocks generate substantially larger inflation and output responses.

Figure 10 shows that the amplification works through both policy accommodation and confusion. For supply shocks, the larger inflation response is closely tied to stronger confusion: the expected demand shock becomes more negative, falling from -0.03 to -0.12. For demand shocks, the amplification is even stronger because two forces move together. First, firms perceive a larger demand shock, with the expected demand shock rising from 0.24 to 0.45. Second, confusion also strengthens: firms infer a much larger negative supply shock, with the expected supply shock falling from -0.21 to -0.46.

The key mechanism is an amplification loop between accommodative policy and inflation through information frictions, especially confusion. When inflation is highly informative, a weaker policy response lets demand shocks generate more inflation. This raises perceived demand after true demand shocks and strengthens negative confusion after both shocks. After supply shocks, negative confusion strengthens because demand shocks now leave a larger inflation footprint, so firms misread more supply-driven inflation as demand-driven. By the same logic, demand shocks induce stronger perceived adverse supply. Because inflation now responds more to both shocks, the inflation signal becomes relatively more

informative, reinforcing own-shock perception and cross-shock confusion. Lenient policy therefore amplifies inflation both directly and through confusion.

The policy implication is that inflation stabilization also stabilizes beliefs. When agents are uncertain about the source of inflation, accommodative policy can make inflation movements informative in the wrong direction, strengthening confusion and amplifying inflation. This is especially relevant in episodes such as the pandemic, when attention to inflation is high but its source is ambiguous. A stronger policy response can therefore dampen inflation directly and limit the confusion-driven amplification.

6 Conclusion

This paper develops a theory of confusion between aggregate demand and supply shocks. When agents infer structural shocks from endogenous aggregates such as output and inflation, they may attribute the same observed movement to the wrong source. The sign and magnitude of this confusion depend on the relative informativeness of the signals agents observe. Confusion also changes the nature of higher-order beliefs and general-equilibrium feedback. Dynamic learning can correct this confusion only when agents observe sufficiently informative signals in both directions.

Quantitatively, the model shows that this mechanism helps organize several features of U.S. inflation dynamics before and after 1990. The estimates point to a shift from an inflation-signal-dominant regime before 1990 to a more output-signal-dominant regime afterward. This change helps explain why supply shocks generated much larger inflation responses of supply shocks before 1990, and why expectations can look supply-driven even when realized fluctuations are demand-driven, and the flattening of the Phillips curve.

For the future directions, three avenues appear especially promising: (i) direct empirical evidence on confusion using targeted surveys; and (ii) faster, more reliable computational methods for solving DSGE models with multiple shocks, endogenous information, and belief distortions at scale.

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Online Appendix to *Confusion and Inflation*

Xiaojie Liu and Dalton Rongxuan Zhang

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A Proofs

A.1 Proposition 1

Proof. It is easy to show that the projection matrix **P** is:

$$\text{Info} = \frac{1}{D}\begin{bmatrix} I_{aa} & I_{ad} \\ I_{ad} & I_{dd} \end{bmatrix}, \tag{A.1}$$

where

$$I_{aa} = \sigma_a^2\left[\sigma_y^2 w_{\pi s}^2 + \sigma_\pi^2 w_{ys}^2 + \sigma_d^2\left(w_{\pi s}w_{yd} + w_{\pi d}w_{ys}\right)^2\right], \tag{A.2}$$

$$I_{ad} = \sigma_a\sigma_d\left[-\sigma_y^2 w_{\pi d}w_{\pi s} + \sigma_\pi^2 w_{yd}w_{ys}\right], \tag{A.3}$$

$$I_{dd} = \sigma_d^2\left[\sigma_y^2 w_{\pi d}^2 + \sigma_\pi^2 w_{yd}^2 + \sigma_a^2\left(w_{\pi s}w_{yd} + w_{\pi d}w_{ys}\right)^2\right]. \tag{A.4}$$

$$\text{D} = \sigma_a^2\sigma_y^2 w_{\pi s}^2 + \sigma_\pi^2\left(\sigma_y^2 + \sigma_d^2 w_{yd}^2 + \sigma_a^2 w_{ys}^2\right) + \sigma_d^2\left(\sigma_y^2 w_{\pi d}^2 + \sigma_a^2(w_{\pi s}w_{yd} + w_{\pi_d}w_{ys})^2\right)$$

Rearrange we get the results. □

A.2 Proposition 2

Proof of Part I. From Proposition 1, we have

$$C(\sigma_y, \sigma_\pi) = \frac{\sigma_a \sigma_d \left(-\sigma_y^2 w_{\pi d} w_{\pi s} + \sigma_\pi^2 w_{yd} w_{ys} \right)}{D(\sigma_y, \sigma_\pi)}, \tag{A.5}$$

$$D(\sigma_y, \sigma_\pi) = \sigma_a^2 \sigma_y^2 w_{\pi s}^2 + \sigma_\pi^2 \left(\sigma_y^2 + \sigma_d^2 w_{yd}^2 + \sigma_a^2 w_{ys}^2 \right) + \sigma_d^2 \left(\sigma_y^2 w_{\pi d}^2 + \sigma_a^2 (w_{\pi s} w_{yd} + w_{\pi d} w_{ys})^2 \right) > 0. \tag{A.6}$$

Fix the relative noise ratio $\rho = \sigma_y^2 / \sigma_\pi^2$, implying $\sigma_\pi^2 = \sigma_y^2 / \rho$. Substituting this relation into D gives

$$\begin{aligned} D(\sigma_y; \rho) &= \sigma_a^2 \sigma_y^2 w_{\pi s}^2 + \frac{\sigma_y^2}{\rho} \left(\sigma_y^2 + \sigma_d^2 w_{yd}^2 + \sigma_a^2 w_{ys}^2 \right) + \sigma_d^2 \left(\sigma_y^2 w_{\pi d}^2 + \sigma_a^2 (w_{\pi s} w_{yd} + w_{\pi d} w_{ys})^2 \right) \\ &= \sigma_y^2 \left[\sigma_a^2 w_{\pi s}^2 + \frac{1}{\rho} (\sigma_d^2 w_{yd}^2 + \sigma_a^2 w_{ys}^2) + \sigma_d^2 w_{\pi d}^2 \right] + \frac{1}{\rho} \sigma_y^4 + \sigma_d^2 \sigma_a^2 (w_{\pi s} w_{yd} + w_{\pi d} w_{ys})^2. \end{aligned} \tag{A.7}$$

Collecting terms by powers of σ_y, we can rewrite (A.7) as

$$D(\sigma_y; \rho) = A\, \sigma_y^2 + B\, \sigma_y^4 + C_0, \tag{A.8}$$

where the constants A, B, and C_0 are

$$A := \sigma_a^2 w_{\pi s}^2 + \frac{1}{\rho} (\sigma_d^2 w_{yd}^2 + \sigma_a^2 w_{ys}^2) + \sigma_d^2 w_{\pi d}^2, \tag{A.9}$$

$$B := \frac{1}{\rho}, \qquad C_0 := \sigma_d^2 \sigma_a^2 (w_{\pi s} w_{yd} + w_{\pi d} w_{ys})^2. \tag{A.10}$$

Substituting back into (A.5), we can factor σ_y^2 from the numerator and obtain

$$C(\sigma_y; \rho) = \frac{\sigma_a \sigma_d\, \sigma_y^2 \left(-w_{\pi d} w_{\pi s} + \frac{w_{yd} w_{ys}}{\rho} \right)}{A\, \sigma_y^2 + B\, \sigma_y^4 + C_0}. \tag{A.11}$$

Limits. As $\sigma_y \to 0$, both numerator and denominator vanish at order σ_y^2, implying

$$\lim_{\sigma_y \to 0} C(\sigma_y; \rho) = 0. \tag{A.12}$$

As $\sigma_y \to \infty$, the denominator grows as $B\sigma_y^4$ while the numerator grows as σ_y^2, giving

$$\lim_{\sigma_y \to \infty} C(\sigma_y; \rho) = 0. \tag{A.13}$$

Sign and shape. The sign of C follows from the numerator in (A.11):

$$\operatorname{sgn}(C) = \operatorname{sgn}\left(-\rho\, w_{\pi d} w_{\pi s} + w_{yd} w_{ys}\right). \tag{A.14}$$

Define the critical ratio

$$\rho^* = \frac{w_{yd} w_{ys}}{w_{\pi d} w_{\pi s}}. \tag{A.15}$$

Then:

- If $\rho < \rho^*$, the numerator of (A.11) is positive, so $C > 0$. Since $C(\sigma_y; \rho)$ behaves as a rational function with quadratic numerator and quartic denominator, it first increases and then decreases in σ_y.
- If $\rho > \rho^*$, the numerator is negative, so $C < 0$, and $C(\sigma_y; \rho)$ first decreases and then increases (a mirrored shape).

Together with (A.12)–(A.13), this proves that

$$\lim_{\sigma_y \to 0} C = \lim_{\sigma_y \to \infty} C = 0,$$

and establishes the shape and sign properties stated in Part I. □

Proof of Part II. Throughout this part, fix the weights

$$\omega := \{w_{\pi d},\, w_{\pi s},\, w_{yd},\, w_{ys}\} > 0$$

For notational clarity, introduce the positive aggregates

$$\Gamma_\pi := \frac{\partial D}{\partial \sigma_\pi^2} = \sigma_y^2 + \sigma_d^2 w_{yd}^2 + \sigma_a^2 w_{ys}^2 \;>\; 0, \tag{A.16}$$

$$\Gamma_y := \frac{\partial D}{\partial \sigma_y^2} = \sigma_a^2 w_{\pi s}^2 + \sigma_\pi^2 + \sigma_d^2 w_{\pi d}^2 \;>\; 0, \tag{A.17}$$

$$K := w_{\pi s} w_{yd} + w_{\pi d} w_{ys}. \tag{A.18}$$

(i) C increases in σ_π for fixed σ_y. Differentiate (A.6) w.r.t. σ_π^2:

$$\frac{\partial C}{\partial \sigma_\pi^2} = \frac{\sigma_a \sigma_d\, w_{yd} w_{ys}\, D - N\, \Gamma_\pi}{D^2}. \tag{A.19}$$

Using $D = \sigma_\pi^2\Gamma_\pi + \underbrace{\sigma_y^2(\sigma_a^2 w_{\pi s}^2 + \sigma_d^2 w_{\pi d}^2) + \sigma_d^2\sigma_a^2 K^2}_{=:R_\pi \geq 0}$ and $N = \sigma_a\sigma_d(-\sigma_y^2 w_{\pi d}w_{\pi s} + \sigma_\pi^2 w_{yd}w_{ys})$, we obtain

$$\begin{aligned}\text{numerator}\left(\frac{\partial C}{\partial \sigma_\pi^2}\right) &= \sigma_a\sigma_d\left[w_{yd}w_{ys}\,(D - \sigma_\pi^2\Gamma_\pi) + \sigma_y^2 w_{\pi d}w_{\pi s}\,\Gamma_\pi\right] \\ &= \sigma_a\sigma_d\left[w_{yd}w_{ys}\,R_\pi + \sigma_y^2 w_{\pi d}w_{\pi s}\,\Gamma_\pi\right]. \end{aligned} \tag{A.20}$$

By construction $R_\pi \geq 0$ and $\Gamma_\pi > 0$. Since both $w_{yd}w_{ys}$ and $w_{\pi d}w_{\pi s}$ are positive; hence the bracket in (A.20) is ≥ 0 and is strictly > 0 unless the weight products are degenerate. We conclude

$$\frac{\partial C}{\partial \sigma_\pi^2} \;>\; 0 \tag{A.21}$$

Thus C is increasing in σ_π for fixed σ_y. Same argument can be applied to prove that C is increasing in σ_y for fixed σ_π. □

A.3 Corollary 1

Proof. Recall the following from Proposition 2:

$$C(\sigma_y, \sigma_\pi) = \frac{\sigma_s\sigma_d\left(-\sigma_y^2\,\omega_{\pi d}\omega_{\pi s} + \sigma_\pi^2\,\omega_{yd}\omega_{ys}\right)}{D(\sigma_y, \sigma_\pi)}, \tag{A.22}$$

$$D(\sigma_y, \sigma_\pi) = \sigma_s^2\sigma_y^2\omega_{\pi s}^2 + \sigma_\pi^2\left(\sigma_y^2 + \sigma_d^2\omega_{yd}^2 + \sigma_s^2\omega_{ys}^2\right) + \sigma_d^2\left(\sigma_y^2\omega_{\pi d}^2 + \sigma_s^2(\omega_{\pi s}\omega_{yd} + \omega_{\pi d}\omega_{ys})^2\right). \tag{A.23}$$

Case 1 (when $C > 0$). Consider the joint limit $\sigma_y \to 0$ and $\sigma_\pi \to \infty$. From (A.22)–(A.23),

$$\text{Num}(\sigma_y, \sigma_\pi) = \sigma_s\sigma_d\left(-\sigma_y^2\,\omega_{\pi d}\omega_{\pi s} + \sigma_\pi^2\,\omega_{yd}\omega_{ys}\right) \;\sim\; \sigma_s\sigma_d\,\sigma_\pi^2\,\omega_{yd}\omega_{ys}, \tag{A.24}$$

$$\text{Den}(\sigma_y, \sigma_\pi) = \underbrace{\sigma_\pi^2\left(\sigma_y^2 + \sigma_d^2\omega_{yd}^2 + \sigma_s^2\omega_{ys}^2\right)}_{\text{order } \sigma_\pi^2} + \underbrace{\sigma_s^2\sigma_y^2\omega_{\pi s}^2 + \sigma_d^2\left(\sigma_y^2\omega_{\pi d}^2 + \sigma_s^2 K^2\right)}_{\text{bounded as } \sigma_\pi \to \infty}, \quad K := \omega_{\pi s}\omega_{yd} + \omega_{\pi d}\omega_{ys}. \tag{A.25}$$

Letting first $\sigma_y \to 0$ and then $\sigma_\pi \to \infty$,

$$\text{Denominator}(\sigma_y, \sigma_\pi) \sim \sigma_\pi^2\left(\sigma_d^2\omega_{yd}^2 + \sigma_s^2\omega_{ys}^2\right). \tag{A.26}$$

Hence

$$\lim_{\sigma_y \to 0,\ \sigma_\pi \to \infty} C(\sigma_y, \sigma_\pi) = \frac{\sigma_s\sigma_d\,\omega_{yd}\omega_{ys}}{\sigma_d^2\omega_{yd}^2 + \sigma_s^2\omega_{ys}^2} =: C_{\max}. \tag{A.27}$$

This proves the first assertion and the closed form for $C_{\max}$.

Case 2 (when $C < 0$). Consider the joint limit $\sigma_\pi \to 0$ and $\sigma_y \to \infty$. From (A.22)–(A.23),

$$\text{Numerator}(\sigma_y, \sigma_\pi) = \sigma_s\sigma_d\Big(-\sigma_y^2\,\omega_{\pi d}\omega_{\pi s} + \sigma_\pi^2\,\omega_{yd}\omega_{ys}\Big) \;\sim\; -\,\sigma_s\sigma_d\,\sigma_y^2\,\omega_{\pi d}\omega_{\pi s}, \tag{A.28}$$

$$\text{Denominator}(\sigma_y, \sigma_\pi) = \underbrace{\sigma_y^2\Big(\sigma_s^2\omega_{\pi s}^2 + \sigma_d^2\omega_{\pi d}^2\Big)}_{\text{order } \sigma_y^2} + \underbrace{\sigma_\pi^2\Big(\sigma_y^2 + \sigma_d^2\omega_{yd}^2 + \sigma_s^2\omega_{ys}^2\Big) + \sigma_d^2\sigma_s^2K^2}_{\text{bounded as } \sigma_y\to\infty}. \tag{A.29}$$

Thus

$$\lim_{\sigma_\pi\to 0,\ \sigma_y\to\infty} C(\sigma_y, \sigma_\pi) = -\frac{\sigma_s\sigma_d\,\omega_{\pi d}\omega_{\pi s}}{\sigma_d^2\omega_{\pi d}^2 + \sigma_s^2\omega_{\pi s}^2} =: C_{\min}. \tag{A.30}$$

This proves the second assertion and the closed form for $C_{\min}$.

Special case ($\sigma_s = \sigma_d$ and all ω's equal to 1). Set $\sigma_s = \sigma_d =: \sigma$ and $\omega_{\pi d} = \omega_{\pi s} = \omega_{yd} = \omega_{ys} = 1$. Then

$$C_{\max} = \frac{\sigma^2}{\sigma^2 + \sigma^2} = \frac{1}{2}, \qquad C_{\min} = -\frac{\sigma^2}{\sigma^2 + \sigma^2} = -\frac{1}{2}, \tag{A.31}$$

so $C_{\max} = -C_{\min} = 1/2$.

Moreover, in the y-dominant limit ($\sigma_y \to 0$, $\sigma_\pi \to \infty$), the limiting *projection matrix* is

$$\mathbf{P}_y = \frac{1}{\sigma_d^2\omega_{yd}^2 + \sigma_s^2\omega_{ys}^2}\begin{pmatrix} \sigma_d^2\omega_{yd}^2 & \sigma_s\sigma_d\omega_{yd}\omega_{ys} \\ \sigma_s\sigma_d\omega_{yd}\omega_{ys} & \sigma_s^2\omega_{ys}^2 \end{pmatrix}. \tag{A.32}$$

With $\sigma_s = \sigma_d = \sigma$ and all $\omega = 1$, this gives

$$\mathbf{P}_y = \frac{1}{2\sigma^2}\begin{pmatrix} \sigma^2 & \sigma^2 \\ \sigma^2 & \sigma^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}. \tag{A.33}$$

Analogously, in the π-dominant limit ($\sigma_\pi \to 0$, $\sigma_y \to \infty$), the limiting projection matrix is

$$\mathbf{P}_\pi = \frac{1}{\sigma_d^2\omega_{\pi d}^2 + \sigma_s^2\omega_{\pi s}^2}\begin{pmatrix} \sigma_d^2\omega_{\pi d}^2 & -\,\sigma_s\sigma_d\omega_{\pi d}\omega_{\pi s} \\ -\,\sigma_s\sigma_d\omega_{\pi d}\omega_{\pi s} & \sigma_s^2\omega_{\pi s}^2 \end{pmatrix}, \tag{A.34}$$

so with $\sigma_s = \sigma_d$ and all $\omega = 1$,

$$\mathbf{P}_\pi = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}. \tag{A.35}$$

□

A.4 Lemma 1

Proof. Let the *projection matrix* be

$$\mathbf{P} = \begin{pmatrix} p_{11} & C \\ C & p_{22} \end{pmatrix} \tag{A.36}$$

Symmetry and positive semidefiniteness; eigenvalue bounds. Let the latent shocks be standardized so that

$$s = (d, a)' \sim N(0, I),$$

and let the vector of observed signals be

$$x = Ws + \varepsilon, \qquad \varepsilon \sim N(0, \Omega), \qquad \Omega \succ 0.$$

The linear-Gaussian projection of shocks on signals implies that the coefficient matrix mapping true shocks into average beliefs is

$$\mathbf{P} = W'(WW' + \Omega)^{-1}W.$$

To show the eigenvalue bounds, whiten the signal noise. Let $B = \Omega^{-1/2}W$. Then

$$\mathbf{P} = B'(I + BB')^{-1}B.$$

Using the identity

$$B'(I + BB')^{-1} = (I + B'B)^{-1}B',$$

we can write

$$\mathbf{P} = (I + B'B)^{-1}B'B.$$

Let

$$M = B'B.$$

Then $M \succeq 0$, and

$$\mathbf{P} = (I + M)^{-1}M = I - (I + M)^{-1}.$$

Therefore $\mathbf{P}$ is symmetric and positive semi-definite. Moreover, $I - \mathbf{P} = (I + M)^{-1} \succeq 0$, so

$$0 \preceq \mathbf{P} \preceq I.$$

Hence all eigenvalues of $\mathbf{P}$ lie in the unit interval:

$$0 \leq \lambda_i \leq 1.$$

Equivalently, if $\mu_i \geq 0$ are the eigenvalues of M, then

$$\lambda_i = \frac{\mu_i}{1 + \mu_i}.$$

Thus $0 \leq \lambda_i < 1$ when signal noise is strictly positive, with $\lambda_i = 1$ obtained only as a limiting case of perfectly precise information.

Spectral decomposition and the $\pm\mu$ structure. Let $\{(\lambda_1, v_1), (\lambda_2, v_2)\}$ be the eigensystem of $\mathbf{P}$, with $0 \le \lambda_2 \le \lambda_1 \le 1$ and $\{v_1, v_2\}$ orthonormal. Then

$$\mathbf{P} = \lambda_1 v_1 v_1' + \lambda_2 v_2 v_2'.$$

Write

$$v_1 = (q, r)', \qquad q^2 + r^2 = 1.$$

An orthonormal vector perpendicular to v_1 can be written as

$$v_2 = (-r, q)'.$$

Define

$$x = q^2, \qquad z = r^2, \qquad \mu = qr.$$

Then

$$v_1 v_1' = \begin{pmatrix} x & \mu \\ \mu & z \end{pmatrix}, \qquad v_2 v_2' = \begin{pmatrix} z & -\mu \\ -\mu & x \end{pmatrix}.$$

Therefore the off-diagonal entry of $\mathbf{P}$, which we call confusion, is

$$C = \left[\lambda_1 v_1 v_1' + \lambda_2 v_2 v_2'\right]_{12} = \lambda_1 \mu - \lambda_2 \mu = (\lambda_1 - \lambda_2)\mu.$$

Thus confusion is the product of two objects: the eigenvalue gap $\lambda_1 - \lambda_2$, which measures how strongly beliefs load on the more informative direction, and the eigen-confusion term μ, which measures how much that informative direction mixes demand and supply.

Eigen-confusion μ is constant for fixed noise ratio. Fix the relative noise ratio $\rho = \frac{\sigma_y^2}{\sigma_\pi^2}$. Scale both signal-noise variances by a common factor $t > 0$, so that

$$\Omega(t; \rho) = t\, \Omega_0(\rho).$$

Using the whitened-noise representation from above, the projection matrix can be written as

$$\mathbf{P}(t; \rho) = B(t)' B(t) \left(I + B(t)' B(t)\right)^{-1},$$

where

$$B(t) = \Omega(t; \rho)^{-1/2} W = t^{-1/2} \Omega_0(\rho)^{-1/2} W.$$

Hence

$$B(t)' B(t) = \frac{1}{t} M_0(\rho), \qquad M_0(\rho) = W' \Omega_0(\rho)^{-1} W.$$

Therefore

$$\mathbf{P}(t; \rho) = \frac{1}{t} M_0(\rho) \left(I + \frac{1}{t} M_0(\rho)\right)^{-1} = M_0(\rho) \left(M_0(\rho) + tI\right)^{-1}.$$

Thus $\mathbf{P}(t; \rho)$ is a matrix function of the fixed symmetric matrix $M_0(\rho)$. It follows that $\mathbf{P}(t; \rho)$ has the same eigenvectors for all $t > 0$. Only the eigenvalues change with t. If v_i are the

eigenvalues of $M_0(\rho)$, then

$$\lambda_i(t) = \frac{\nu_i}{\nu_i + t}.$$

Therefore, for fixed ω, shock process, and noise ratio ρ, the eigen-confusion term μ is constant in the common noise scale t. Common increases in signal noise shrink the eigenvalues, but do not rotate the eigenvectors.

□

A.5 Proposition 3

Proof. Let $s = (\beta, a)'$. Since first-order average beliefs are linear, $\bar{E}^1 s = \mathbf{P}s$, higher-order beliefs are obtained by iterating the same projection: $\bar{E}^h s = \mathbf{P}^h s$.

By Proposition 1, $\mathbf{P}$ is symmetric positive semidefinite, with

$$\mathbf{P} = \lambda_1 v_1 v_1' + \lambda_2 v_2 v_2', \qquad 0 \le \lambda_2 \le \lambda_1 \le 1.$$

Hence

$$\mathbf{P}^h = \lambda_1^h v_1 v_1' + \lambda_2^h v_2 v_2'.$$

Taking the off-diagonal entry gives

$$C_h = (\mathbf{P}^h)_{12} = (\lambda_1^h - \lambda_2^h)\mu.$$

In the nondegenerate case $0 < \lambda_2 < \lambda_1 < 1$, define $f(h) = \lambda_1^h - \lambda_2^h$. Then $f(0) = 0$, $\lim_{h\to\infty} f(h) = 0$, and

$$f'(h) = \lambda_1^h \log \lambda_1 - \lambda_2^h \log \lambda_2.$$

Since $\log \lambda_i < 0$, $f'(h) > 0$ for small h and $f'(h) < 0$ for large h. The unique maximizer is

$$h^\star = \frac{\log(\log \lambda_2 / \log \lambda_1)}{\log(\lambda_1 / \lambda_2)}.$$

Thus $f(h)$, and therefore $|C_h| = |\mu| f(h)$, first rises and then falls with the order of beliefs.

In the one-signal limits, $\lambda_1 \to 1$. If output is perfectly precise, then

$$C_h \to \mu_y(1 - \lambda_2^h), \qquad \lim_{h\to\infty} C_h = C_{\max}.$$

If inflation is perfectly precise, then

$$C_h \to \mu_\pi(1 - \lambda_2^h), \qquad \lim_{h\to\infty} C_h = C_{\min}.$$

□

A.6 Proposition 4

The following lemma shows that both the covariance matrix P and the one-step projection KH share the same eigenvectors as the information matrix.

Lemma A.1 (Commutation of the Kalman projection)**.** *Let*

$$M := H^\top R^{-1} H.$$

Under $F = \rho I_2$ and $Q = \sigma_u^2 I_2$, the steady-state prior covariance Σ and the one-step projection $G := KH$ share the eigenvectors of M. That is, if

$$M = U \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} U^\top,$$

then

$$\Sigma = U \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix} U^\top, \qquad G = U \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} U^\top,$$

where $p_i > 0$ solves

$$m_i p_i^2 + \left[(1-\rho^2) - \sigma_u^2 m_i\right] p_i - \sigma_u^2 = 0,$$

and

$$\alpha_i = \frac{p_i m_i}{1 + p_i m_i} \in [0, 1).$$

Proof. Rotate states by U, so that

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}.$$

Since $F = \rho I_2$ and $Q = \sigma_u^2 I_2$, this rotation leaves the state dynamics unchanged. Let Σ denote the steady-state prior covariance. The Riccati equation is

$$\Sigma = \rho^2 \left[\Sigma - \Sigma H^\top (H \Sigma H^\top + R)^{-1} H \Sigma\right] + \sigma_u^2 I_2.$$

By the Woodbury identity,

$$\Sigma - \Sigma H^\top (H \Sigma H^\top + R)^{-1} H \Sigma = (\Sigma^{-1} + H^\top R^{-1} H)^{-1} = (\Sigma^{-1} + M)^{-1}.$$

Therefore, in the rotated basis,

$$\Sigma = \rho^2 (\Sigma^{-1} + M)^{-1} + \sigma_u^2 I_2.$$

Because M is diagonal, this equation preserves diagonality. By uniqueness of the stabilizing solution,

$$\Sigma = \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix}.$$

The i-th diagonal entry satisfies

$$p_i = \rho^2 \frac{p_i}{1 + p_i m_i} + \sigma_u^2,$$

which is equivalent to

$$m_i p_i^2 + \left[(1-\rho^2) - \sigma_u^2 m_i\right] p_i - \sigma_u^2 = 0.$$

Finally,

$$G = KH = \Sigma H^\top (H\Sigma H^\top + R)^{-1} H.$$

Using the same algebra,

$$G = \Sigma M (I + \Sigma M)^{-1}.$$

Since Σ and M are diagonal in the same basis,

$$G = \begin{pmatrix} \dfrac{p_1 m_1}{1 + p_1 m_1} & 0 \\ 0 & \dfrac{p_2 m_2}{1 + p_2 m_2} \end{pmatrix}.$$

Thus, after transforming back by U,

$$G = U \begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{pmatrix} U^\top, \qquad \alpha_i = \frac{p_i m_i}{1 + p_i m_i}.$$

This proves that M, Σ, and $G = KH$ share the same eigenvectors. □

Now we can prove the proposition:

Proof. Let $m_t := \bar{E} s_t$ and define $G := KH$. Averaging over idiosyncratic signal noise, the steady-state Kalman filter gives

$$m_t = (I - G) F m_{t-1} + G s_t.$$

Since $F = \rho I_2$,

$$m_t = \rho (I - G) m_{t-1} + G s_t.$$

By Lemma A.1, G is symmetric. Write

$$G = \begin{pmatrix} D_d & C \\ C & D_s \end{pmatrix}.$$

Then

$$I - G = \begin{pmatrix} 1 - D_d & -C \\ -C & 1 - D_s \end{pmatrix}.$$

Substituting $m_t = (\bar{E}\beta_t, \bar{E} a_t)^\top$ and $s_t = (\beta_t, a_t)^\top$ gives

$$\bar{E}\beta_t = \rho(1 - D_d)\bar{E}\beta_{t-1} + D_d \beta_t + C(a_t - \rho \bar{E} a_{t-1}),$$

and

$$\bar{E} a_t = \rho(1 - D_s)\bar{E} a_{t-1} + D_s a_t + C(\beta_t - \rho \bar{E}\beta_{t-1}).$$

□

A.7 Corollary 3

Proof. The Kalman filter satisfies

$$m_t = \rho(I - G)m_{t-1} + Gs_t.$$

Define

$$e_+ := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \qquad e_- := \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}.$$

First suppose the inflation signal is uninformative. In the symmetric case, the only informative direction is e_+. Hence

$$G = \alpha_1 e_+ e_+', \qquad \alpha_1 \in (0,1).$$

Projecting the belief law onto e_+ and e_- gives

$$e_+' m_t = \rho(1-\alpha_1)e_+' m_{t-1} + \alpha_1 e_+' s_t,$$

and

$$e_-' m_t = \rho e_-' m_{t-1}.$$

Thus the unobserved difference component is only a transient. With zero initial belief in that direction, or in the stationary limit, $e_-' m_t = 0$. Therefore

$$e_+' m_t = \frac{\alpha_1}{1-\rho(1-\alpha_1)L} e_+' s_t.$$

Since $e_+' s_t = (\beta_t + a_t)/\sqrt{2}$ and $m_t = (e_+' m_t)e_+$, we obtain

$$\bar{E}\beta_t = \bar{E}a_t = \frac{1}{2}\frac{\alpha_1}{1-\rho(1-\alpha_1)L}(\beta_t + a_t).$$

Now suppose the output signal is uninformative. The only informative direction is e_-, so

$$G = \alpha_1 e_- e_-', \qquad \alpha_1 \in (0,1).$$

Projecting onto e_- and e_+ gives

$$e_-' m_t = \rho(1-\alpha_1)e_-' m_{t-1} + \alpha_1 e_-' s_t,$$

and

$$e_+' m_t = \rho e_+' m_{t-1}.$$

Again, the unobserved sum component is only a transient. With zero initial belief in that direction, or in the stationary limit,

$$e_-' m_t = \frac{\alpha_1}{1-\rho(1-\alpha_1)L} e_-' s_t.$$

Since $e'_- s_t = (\beta_t - a_t)/\sqrt{2}$ and $m_t = (e'_- m_t)e_-$, it follows that

$$\bar{E}\beta_t = -\bar{E}a_t = \frac{1}{2}\frac{\alpha_1}{1-\rho(1-\alpha_1)L}(\beta_t - a_t).$$

□

A.8 Proposition 5

Proof. **Step 0. Notation and belief operator.** Let $s_t := (\beta_t, a_t)^\top$ collect demand and supply shocks. Write the linear "belief" (average projection) as

$$\bar{E}[s_t] = \mathbf{P}\, s_t, \qquad \mathbf{P} = \begin{pmatrix} D_d & C \\ C & D_s \end{pmatrix},$$

so that, under a unit *demand* shock ($\beta = 1, a = 0$), $\bar{E}[\beta] = D_d$, $\bar{E}[a] = C$; under a unit *supply* shock ($\beta = 0, a = 1$), $\bar{E}[\beta] = C$, $\bar{E}[a] = D_s$.

We seek responses $\pi_t = \tilde{\pi}_d\, \beta_t + \tilde{\pi}_s\, a_t$, $y_t = \tilde{y}_d\, \beta_t + \tilde{y}_s\, a_t$, period by period.

Step 1. Supply (pricing) equation under each unit shock. The pricing rule is

$$\pi_t \;=\; \bar{E}(\pi_t + \psi_y y_t) \;-\; \psi_a(\eta\, \bar{E}a_t + a_t).$$

(i) Demand shock: with $(\beta, a) = (1, 0)$,

$$\tilde{\pi}_d = D_d(\tilde{\pi}_d + \psi_y \tilde{y}_d) + C(\tilde{\pi}_s + \psi_y \tilde{y}_s) \;-\; \psi_a(\eta \underbrace{\bar{E}a}_{=C} + 0),$$

which rearranges to

$$(1 - D_d)\tilde{\pi}_d \;=\; \psi_y D_d \tilde{y}_d + C(\tilde{\pi}_s + \psi_y \tilde{y}_s) - \eta\psi_a C. \tag{A.37}$$

(ii) Supply shock: with $(\beta, a) = (0, 1)$,

$$\tilde{\pi}_s = C(\tilde{\pi}_d + \psi_y \tilde{y}_d) + D_s(\tilde{\pi}_s + \psi_y \tilde{y}_s) \;-\; \psi_a(\eta \underbrace{\bar{E}a}_{=D_s} + 1),$$

i.e.

$$(1 - D_s)\tilde{\pi}_s \;=\; \psi_y D_s \tilde{y}_s + C(\tilde{\pi}_d + \psi_y \tilde{y}_d) - \eta\psi_a D_s - \psi_a. \tag{A.38}$$

Step 2. IS/Euler equation and output responses. With a (contemporaneous) Taylor rule $i_t = \phi\pi_t$, the Euler equation $y_t = y_{t+1} - \frac{1}{\sigma}(i_t - \pi_{t+1}) + \beta_t$ implies a standard linear solution

$$\tilde{y}_d = -\frac{\phi}{\sigma}\,\tilde{\pi}_d + F_d, \qquad F_d \;=\; \frac{\phi}{\phi - \rho_d}, \tag{A.39}$$

$$\tilde{y}_s = -\frac{\phi}{\sigma}\,\tilde{\pi}_s + F_s, \qquad F_s \;=\; (1+\eta)\,\frac{\psi_a}{\psi_y}\,\rho_s\,\frac{\phi - 1}{\phi - \rho_s}, \tag{A.40}$$

where ρ_d, ρ_s are the AR(1) persistence parameters of (β_t, a_t) and the constants F_d, F_s collect the direct intertemporal (forward-looking) effects of the shocks when households know the shocks and the fact that firms will know the shocks as well in the next period. □

A.9 Theorem 1

Proof. Let

$$\mathbf{P} = \begin{pmatrix} D_d & C \\ C & D_s \end{pmatrix}, \qquad \tilde{\boldsymbol{\pi}} = \begin{pmatrix} \tilde{\pi}_d \\ \tilde{\pi}_s \end{pmatrix}.$$

Substitute

$$\tilde{y}_d = -\frac{\phi}{\sigma}\tilde{\pi}_d + F_d, \qquad \tilde{y}_s = -\frac{\phi}{\sigma}\tilde{\pi}_s + F_s$$

into the first pricing equation. Multiplying by $1 - D_d$ gives

$$(1 - D_d)\tilde{\pi}_d = \psi_y D_d \left(-\frac{\phi}{\sigma}\tilde{\pi}_d + F_d\right) + C\tilde{\pi}_s + \psi_y C \left(-\frac{\phi}{\sigma}\tilde{\pi}_s + F_s\right) - \eta\psi_a C.$$

Collecting terms,

$$(1 - \delta D_d)\tilde{\pi}_d - \delta C\tilde{\pi}_s = \psi_y(D_d F_d + C F_s) - \eta\psi_a C = U_d,$$

where

$$\delta = 1 - \frac{\phi\psi_y}{\sigma}.$$

Similarly, the second pricing equation gives

$$-\delta C\tilde{\pi}_d + (1 - \delta D_s)\tilde{\pi}_s = \psi_y(C F_d + D_s F_s) - \eta\psi_a D_s - \psi_a = U_s.$$

Therefore,

$$(\mathbf{I} - \delta\mathbf{P})\tilde{\boldsymbol{\pi}} = \mathbf{U},$$

and hence

$$\tilde{\boldsymbol{\pi}} = (\mathbf{I} - \delta\mathbf{P})^{-1}\mathbf{U}.$$

Now suppose $|\delta| < 1$. Since the eigenvalues of $\mathbf{P}$ satisfy $0 \le \lambda_k \le 1$, the inverse exists and

$$(\mathbf{I} - \delta\mathbf{P})^{-1} = \mathbf{I} + \delta\mathbf{P}(\mathbf{I} - \delta\mathbf{P})^{-1}.$$

Using the spectral decomposition

$$\mathbf{P} = \lambda_1 v_1 v_1^\top + \lambda_2 v_2 v_2^\top,$$

we have

$$\delta\mathbf{P}(\mathbf{I} - \delta\mathbf{P})^{-1} = \tilde{\lambda}_1 v_1 v_1^\top + \tilde{\lambda}_2 v_2 v_2^\top, \qquad \tilde{\lambda}_k = \frac{\delta\lambda_k}{1 - \delta\lambda_k}.$$

Write

$$v_1 v_1^\top = \begin{pmatrix} x & \mu \\ \mu & z \end{pmatrix}, \qquad v_2 v_2^\top = \begin{pmatrix} z & -\mu \\ -\mu & x \end{pmatrix}.$$

Then

$$\delta \mathbf{P}(\mathbf{I} - \delta \mathbf{P})^{-1} = \begin{pmatrix} \tilde{\lambda}_1 x + \tilde{\lambda}_2 z & (\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu \\ (\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu & \tilde{\lambda}_1 z + \tilde{\lambda}_2 x \end{pmatrix}.$$

Thus

$$\tilde{\boldsymbol{\pi}} = \mathbf{U} + \delta \mathbf{P}(\mathbf{I} - \delta \mathbf{P})^{-1}\mathbf{U},$$

which implies

$$\tilde{\boldsymbol{\pi}} = \underbrace{\begin{pmatrix} U_d \\ U_s \end{pmatrix}}_{\text{PE}} + \underbrace{\begin{pmatrix} (\tilde{\lambda}_1 x + \tilde{\lambda}_2 z)U_d \\ (\tilde{\lambda}_1 z + \tilde{\lambda}_2 x)U_s \end{pmatrix}}_{\text{GE: own-shock}} + \underbrace{(\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu \begin{pmatrix} U_s \\ U_d \end{pmatrix}}_{\text{GE: confusion}}.$$

This proves the result. □

A.10 Sign of Confusion

There are confusion inside both PE and GE inflation responses. The following corollary characterizes when PE and GE confusion amplify inflation responses.

Corollary A.1 (Sign of PE and GE Confusion Effects)**.** *Suppose $U_d > 0$, $U_s < 0$, and $\delta > 0$. Let*

$$\bar{\rho}_a = \frac{\eta\phi}{(1+\eta)\phi - 1}.$$

(i) Supply shocks. *For a positive supply shock, PE confusion and GE confusion move in the same direction. If $C < 0$, both amplify the disinflationary response. If $C > 0$, both dampen the disinflationary response.*

(ii) Demand shocks. *For a positive demand shock, the GE confusion effect is unambiguous: it amplifies the inflation response when $C < 0$ and dampens it when $C > 0$. The PE confusion effect depends on supply-shock persistence. If $\rho_a < \bar{\rho}_a$, PE confusion moves in the same direction as GE confusion. If $\rho_a > \bar{\rho}_a$, PE confusion offsets GE confusion.*

Proof. Let

$$\Gamma := (\tilde{\lambda}_1 - \tilde{\lambda}_2)\mu.$$

Since $\delta > 0$, the map

$$\lambda \mapsto \frac{\delta\lambda}{1 - \delta\lambda}$$

is increasing on the relevant range. Hence

$$\operatorname{sign}(\Gamma) = \operatorname{sign}\big((\lambda_1 - \lambda_2)\mu\big) = \operatorname{sign}(C),$$

where $C = (\lambda_1 - \lambda_2)\mu$.

From Theorem 1, the GE confusion terms are

$$\Gamma U_s \quad \text{for demand shocks,} \qquad \Gamma U_d \quad \text{for supply shocks.}$$

Since $U_d > 0$ and $U_s < 0$, the GE confusion effect has sign C for supply shocks and sign $-C$ for demand shocks. Thus, for supply shocks, $C < 0$ makes the GE term negative and amplifies disinflation, while $C > 0$ makes it positive and dampens disinflation. For demand shocks, $C < 0$ makes the GE term positive and amplifies inflation, while $C > 0$ makes it negative and dampens inflation.

It remains to characterize the PE confusion terms. From the definition of $\mathbf{U}$,

$$U_d = \psi_y D_d F_d + C(\psi_y F_s - \eta \psi_a),$$

so the PE confusion term for a demand shock is

$$C(\psi_y F_s - \eta \psi_a).$$

Similarly,

$$U_s = \psi_y D_s F_s - \eta \psi_a D_s - \psi_a + C\psi_y F_d,$$

so the PE confusion term for a supply shock is

$$C\psi_y F_d.$$

Because $\psi_y > 0$ and $F_d > 0$, the supply-shock PE confusion term has sign C. Hence PE and GE confusion move in the same direction for supply shocks: $C < 0$ amplifies disinflation, while $C > 0$ dampens it.

For demand shocks, use

$$F_s = (1+\eta)\frac{\psi_a}{\psi_y}\rho_a \frac{\phi - 1}{\phi - \rho_a}.$$

Then

$$\psi_y F_s - \eta \psi_a = \psi_a \left[(1+\eta)\rho_a \frac{\phi - 1}{\phi - \rho_a} - \eta \right].$$

Since $\psi_a > 0$ and $\phi - \rho_a > 0$, the sign is determined by

$$(1+\eta)\rho_a(\phi - 1) - \eta(\phi - \rho_a) = \rho_a\big((1+\eta)\phi - 1\big) - \eta\phi.$$

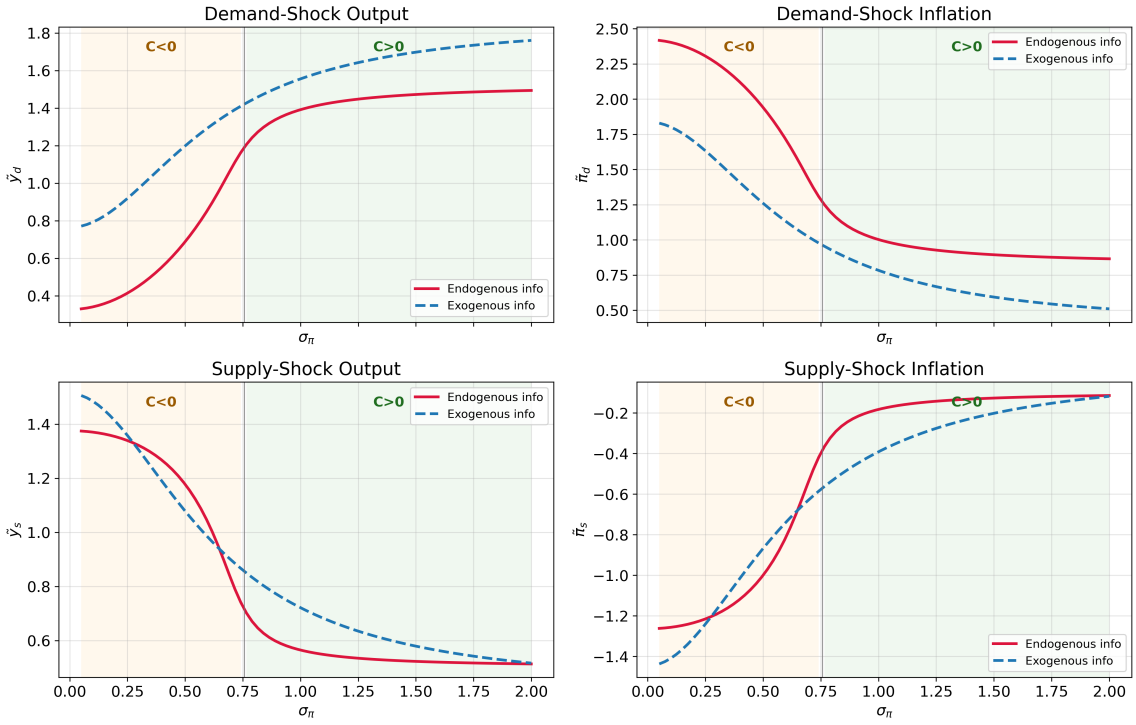
Therefore

$$\psi_y F_s - \eta \psi_a \begin{cases} < 0, & \rho_a < \bar{\rho}_a, \\ = 0, & \rho_a = \bar{\rho}_a, \\ > 0, & \rho_a > \bar{\rho}_a, \end{cases} \qquad \bar{\rho}_a = \frac{\eta\phi}{(1+\eta)\phi - 1}.$$

Thus, when $\rho_a < \bar{\rho}_a$, the PE demand-confusion term has sign $-C$, the same sign as the GE demand-confusion term. When $\rho_a > \bar{\rho}_a$, it has sign C, and therefore offsets the GE confusion effect. This proves the corollary. □

The corollary highlights that for supply shocks, confusion has a sharp sign. When $C < 0$, both the PE confusion term and the GE confusion term push inflation downward. For demand shocks, the GE confusion effect is also sharply signed because it solely depends on U_s. Under $U_s < 0$, negative confusion $C < 0$ makes the GE confusion term raise inflation. The PE confusion effect, however, depends on whether the mistaken supply-shock

Figure A.1: Effect of Confusion on Output and Inflation with Endogenous Information



Notes: The figure plots inflation and output responses to demand and supply shocks as the noise in the inflation signal, σ_π, varies. Solid lines are responses under endogenous information, where signal loadings are consistent with equilibrium responses. Dashed lines are responses under exogenous information, where signal loadings are fixed.

interpretation raises or lowers expected real marginal cost. If the supply shock is less persistent, the productivity component dominates and PE confusion raises inflation under $C < 0$. If the supply shock is more persistent, the real-wage component dominates and PE confusion lowers inflation under $C < 0$.

A.11 Propositions in Section 3.2

We first consider the case where there is only demand shock and firms only receive signals about inflation. The equations are:

$$y_t = -(\phi/\sigma)\,\pi_t + F_d\beta_t \tag{A.41}$$

$$\pi_t = \bar{E}(\pi_t + \psi_y y_t) \tag{A.42}$$

The following proposition shows the equilibrium multiplicity in this setup.

Proposition A.1 (Demand Only with an Inflation Signal)**.** *Suppose firms observe only*

$$x_{\pi t} = \pi_t + \varepsilon_{\pi t}, \qquad Var(\varepsilon_{\pi t}) = \sigma_\pi^2,$$

and let $Var(\beta_t) = \sigma_d^2 > 0$. In any linear RE equilibrium $\pi_t = \tilde{\pi}_d \beta_t$, the inflation response satisfies

$$\tilde{\pi}_d \in \left\{ 0,\ \frac{\sigma F_d}{2\phi} \left[1 \pm \sqrt{1 - \frac{4\phi\sigma_\pi^2}{\sigma\sigma_d^2\psi_y F_d^2}} \right] \right\}.$$

The two nonzero equilibria exist iff

$$\sigma_\pi^2 \leq \frac{\sigma_d^2 \sigma \psi_y F_d^2}{4\phi}.$$

If this condition fails, the unique equilibrium is $\tilde{\pi}_d = 0$.

Proof. Let $\pi_t = T\beta_t$, $b = \phi/\sigma$, and $\gamma = \phi\psi_y/\sigma$. Then

$$\pi_t + \psi_y y_t = \big[(1-\gamma)T + \psi_y F_d\big]\beta_t.$$

With only the inflation signal, the posterior gain on β_t is

$$\omega(T) = \frac{\sigma_d^2 T^2}{\sigma_d^2 T^2 + \sigma_\pi^2}.$$

The pricing equation is therefore

$$T = \omega(T)\big[(1-\gamma)T + \psi_y F_d\big].$$

After clearing denominators,

$$T\left(\gamma\sigma_d^2 T^2 - \psi_y F_d \sigma_d^2 T + \sigma_\pi^2\right) = 0.$$

Thus $T = 0$, or T solves the stated quadratic. The discriminant condition gives the existence threshold for the two nonzero roots. □

Proposition A.2 (Demand Only with Inflation and Output Signals)**.** *With two signals, in the demand-shock only model, any linear RE equilibrium $T = \tilde{\pi}_d$ satisfies*

$$\gamma\sigma_d^2 T^3 - \psi_y F_d \sigma_d^2 T^2 + \sigma_\pi^2 T = \frac{\sigma_\pi^2}{\sigma_y^2}\psi_y \sigma_d^2 (F_d - bT)^3, \tag{A.43}$$

As $\sigma_y \to \infty$, this equation converges to the one-signal cubic.

Define

$$\bar{\sigma}_\pi^2 := \frac{\sigma_d^2 \sigma \psi_y F_d^2}{3\phi}.$$

If $\sigma_\pi^2 \geq \bar{\sigma}_\pi^2$, the equilibrium is unique for any σ_y. If $\sigma_\pi^2 < \bar{\sigma}_\pi^2$, the equilibrium is unique whenever

$$\sigma_y^2 \leq \left(\frac{\phi}{\sigma}\right)^2 \sigma_\pi^2 \frac{\sigma_\pi^2 + 3\bar{\sigma}_\pi^2}{\bar{\sigma}_\pi^2 - \sigma_\pi^2}.$$

Proof. Let $\pi_t = T\beta_t$. Then

$$y_t = (F_d - bT)\beta_t, \qquad \pi_t + \psi_y y_t = \big[(1-\gamma)T + \psi_y F_d\big]\beta_t.$$

The two-signal posterior gain on β_t is

$$K(T) = \frac{q(T)}{\sigma_d^{-2} + q(T)}, \qquad q(T) = \frac{T^2}{\sigma_\pi^2} + \frac{(F_d - bT)^2}{\sigma_y^2}.$$

The pricing equation

$$T = K(T)\big[(1-\gamma)T + \psi_y F_d\big]$$

is equivalent to (A.43). It remains to derive the uniqueness condition. Write

$$H(T) = \gamma\sigma_d^2 T^3 - \psi_y F_d \sigma_d^2 T^2 + \sigma_\pi^2 T - \frac{\sigma_\pi^2}{\sigma_y^2}\psi_y \sigma_d^2 (F_d - bT)^3.$$

Then

$$H'(T) = \sigma_\pi^2 + \psi_y \sigma_d^2 \left[3bT^2 - 2F_d T + 3\frac{\sigma_\pi^2}{\sigma_y^2} b(F_d - bT)^2\right].$$

Let

$$m := \frac{b^2\sigma_\pi^2}{\sigma_y^2}.$$

The quadratic term inside brackets can be written as

$$3b(1+m)T^2 - 2F_d(1+3m)T + \frac{3mF_d^2}{b}.$$

Its minimum is attained at

$$T^\star = \frac{F_d(1+3m)}{3b(1+m)}.$$

Evaluating at $T^\star$ gives

$$\min_T \left[3bT^2 - 2F_d T + 3\frac{\sigma_\pi^2}{\sigma_y^2} b(F_d - bT)^2\right] = \frac{F_d^2}{b}\frac{3m-1}{3(1+m)}.$$

Therefore

$$\min_T H'(T) = \sigma_\pi^2 + \frac{\sigma_d^2 \sigma \psi_y F_d^2}{3\phi}\frac{3m-1}{1+m}.$$

Define

$$\bar{\sigma}_\pi^2 := \frac{\sigma_d^2 \sigma \psi_y F_d^2}{3\phi}.$$

Thus $H'(T) \geq 0$ for all T whenever

$$\sigma_\pi^2 + \bar{\sigma}_\pi^2 \frac{3m-1}{1+m} \geq 0.$$

If $\sigma_\pi^2 \geq \bar{\sigma}_\pi^2$, this holds for any $m \geq 0$, hence for any σ_y. If $\sigma_\pi^2 < \bar{\sigma}_\pi^2$, the condition is equivalent to

$$m \geq \frac{\bar{\sigma}_\pi^2 - \sigma_\pi^2}{\sigma_\pi^2 + 3\bar{\sigma}_\pi^2}.$$

Since $m = b^2\sigma_\pi^2/\sigma_y^2$, this becomes

$$\sigma_y^2 \leq b^2\sigma_\pi^2 \frac{\sigma_\pi^2 + 3\bar{\sigma}_\pi^2}{\bar{\sigma}_\pi^2 - \sigma_\pi^2}.$$

Under this condition H is increasing, so the equilibrium equation $H(T) = 0$ has a unique solution.

□

Now, consider the pricing system

$$\tilde{\pi}_d = (1 - D_d)^{-1}\Big(\psi_y D_d \tilde{y}_d + C(\tilde{\pi}_s + \psi_y \tilde{y}_s) - \eta\psi_a C\Big),$$
$$\tilde{\pi}_s = (1 - D_s)^{-1}\Big(\psi_y D_s \tilde{y}_s + C(\tilde{\pi}_d + \psi_y \tilde{y}_d) - \eta\psi_a D_s - \psi_a\Big),$$

where $\tilde{y}_d = -(\phi/\sigma)\tilde{\pi}_d + F_d$ and $\tilde{y}_s = -(\phi/\sigma)\tilde{\pi}_s + F_s$.

Proposition A.3 (Two-Shock One-Signal Equilibrium.)**.** *Suppose firms observe only the inflation signal*

$$x_{\pi t} = \pi_t + \varepsilon_{\pi t}.$$

Then there is an equilibrium in which

$$\tilde{\pi}_d = 0, \qquad C = 0,$$

and the supply-shock response solves the scalar fixed point

$$\tilde{\pi}_s = (1 - D_s)^{-1}\left(\psi_y D_s \tilde{y}_s - \eta\psi_a D_s - \psi_a\right), \qquad \tilde{y}_s = -(\phi/\sigma)\tilde{\pi}_s + F_s.$$

If the demand-only inflation-signal fixed point admits a nonzero demand response, then there is also an equilibrium with

$$\tilde{\pi}_d \neq 0.$$

In that equilibrium the inflation signal loads on both shocks, so firms confuse demand and supply shocks and generically

$$C \neq 0.$$

Proof. Set $\tilde{\pi}_d = 0$. Then the inflation signal is driven only by the supply shock:

$$x_{\pi t} = \tilde{\pi}_s \beta_{st} + \varepsilon_{\pi t}.$$

Since the demand shock is orthogonal to the signal, the posterior loading on the demand shock is zero. Therefore

$$D_d = 0, \qquad C = 0.$$

The demand pricing equation then gives $\tilde{\pi}_d = 0$ immediately. The remaining supply equation is one-dimensional and gives the stated scalar fixed point.

In this equilibrium, the inflation signal contains no information about the demand shock. Hence firms attribute inflation movements entirely to the supply shock, and the pricing system collapses to the demand-only, inflation-signal case for the demand block.

If instead the demand-only inflation-signal problem has a nonzero solution $\tilde{\pi}_d \neq 0$, then the same inflation signal loads on both shocks:

$$x_{\pi t} = \tilde{\pi}_d \beta_{dt} + \tilde{\pi}_s \beta_{st} + \varepsilon_{\pi t}.$$

The posterior projection is no longer block diagonal. Thus the off-diagonal confusion term is generically nonzero, $C \neq 0$, giving a distinct equilibrium with confusion. □

Figure A.1 illustrates how endogenous information shapes the inflation and output responses. The inflation response to a demand shock is markedly larger than under exogenous signal weights. The mechanism is self-reinforcing. Starting from the exogenous-information equilibrium, demand shocks already generate a sizable inflation response. Endogenizing information therefore increases the demand loading in the inflation signal, as implied by equation (2). This makes demand shocks easier to identify from inflation, induces firms to adjust prices more strongly, and further amplifies the inflation response.

This stark amplification is partly specific to the simple model, where demand shocks generate large inflation movements. In the quantitative model with nominal rigidities, demand-driven inflation responses are more muted. Nevertheless, the underlying mechanism is general: endogenous information makes agents learn more from shocks that move observable aggregates more strongly. If inflation is primarily supply-driven, as under a flat Phillips curve, then inflation becomes a better signal of supply shocks and confusion about supply is limited. Output, by contrast, is typically driven by both demand and supply, so output-based learning can generate sizable confusion in both directions.

B Additional Details on Calibration and Estimation

B.1 Deriving the wage equation and flexible-price target

This section derives the habit-adjusted wage equation and the corresponding flexible-price target used in the quantitative model. Households have external habit in consumption. Log-linearizing the household labor-supply condition gives

$$w_{ikt} - p_{it} = \sigma \left(\frac{1}{1-\lambda_c} c_{it} - \frac{\lambda_c}{1-\lambda_c} c_{i,t-1} \right) + \eta n_{ikt}. \tag{B.1}$$

Let

$$\tilde{c}^h_{it} \equiv \frac{1}{1-\lambda_c} c_{it} - \frac{\lambda_c}{1-\lambda_c} c_{i,t-1}.$$

The labor union aggregates differentiated labor services according to

$$N_{kt} = \left(\int_{\Omega_k} N_{ikt}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}},$$

which implies the demand curve for household i's labor:

$$n_{ikt} = n_{kt} - \epsilon_w (w_{ikt} - w_{kt}).$$

Substituting this demand curve into (B.1) gives

$$(1 + \eta \epsilon_w) w_{ikt} = p_{it} + \sigma \tilde{c}_{it}^h + \eta n_{kt} + \eta \epsilon_w w_{kt}.$$

Aggregating across households within union k, and using the log-linear wage index, yields

$$w_{kt} = p_{kt}^H + \sigma \tilde{c}_{kt}^h + \eta n_{kt},$$

where p_{kt}^H and $\tilde{c}_{kt}^h$ denote union-level averages. Aggregating once more across unions gives the aggregate wage equation

$$w_t - p_t = \frac{\sigma}{1 - \lambda_c} (c_t - \lambda_c c_{t-1}) + \eta n_t. \tag{B.2}$$

Using goods-market clearing $c_t = y_t$ and aggregate labor demand,

$$n_t = \frac{1}{1 - \alpha} (y_t - a_t),$$

equation (B.2) implies

$$w_t = p_t + \frac{\sigma}{1 - \lambda_c} (y_t - \lambda_c y_{t-1}) + \frac{\eta}{1 - \alpha} (y_t - a_t).$$

The firm's flexible-price target is

$$p_{jt}^f = \mathbb{E}_{jt} \left[\varphi \left(p_t + \frac{1}{\varepsilon} y_t \right) + (1 - \varphi) w_t \right] - \frac{1 - \varphi}{1 - \alpha} a_{jt}.$$

Substituting the wage equation into this expression and collecting terms gives

$$p_{jt}^f = \mathbb{E}_{jt} \left[p_t + \psi_y' y_t - \psi_\lambda y_{t-1} - \eta \psi_a a_t \right] - \psi_a a_{jt}, \tag{B.3}$$

where

$$\psi_y' \equiv \frac{\varphi}{\varepsilon} + (1 - \varphi) \left(\frac{\sigma}{1 - \lambda_c} + \frac{\eta}{1 - \alpha} \right), \qquad \psi_\lambda \equiv (1 - \varphi) \frac{\sigma \lambda_c}{1 - \lambda_c}, \qquad \psi_a \equiv \frac{1 - \varphi}{1 - \alpha}.$$

Finally, under Calvo pricing, the standard log-linear reset-price equation is

$$z_{jt} = \mathbb{E}_{jt}\pi_t + (1 - \beta\theta)\left(p_{jt}^f - p_t\right) + \beta\theta\mathbb{E}_{jt}z_{j,t+1}.$$

Combining this expression with (B.3) gives equation (18) in the main text.

B.2 Computational details

Relative to the baseline implementation of the analytic policy-function iteration method, our solver introduces three numerical refinements that improve speed and robustness while leaving the model and equilibrium concept unchanged.

First, the baseline algorithm updates the policy function through damped fixed-point iteration. While simple and robust, this procedure can be slow and may exhibit oscillatory behavior when the fixed-point mapping is only weakly contractive. We therefore augment the iteration with a safeguarded acceleration step that uses information from several recent iterates to construct an improved candidate update. To preserve numerical stability, this candidate is accepted only when it improves the convergence diagnostics relative to the baseline damped update. This refinement reduces the number of iterations required for convergence in many cases and makes the solution procedure more reliable in difficult regions of the parameter space.

Second, in incomplete-information models, a substantial share of the computational burden arises from repeatedly constructing conditional laws of motion and forecasting objects across multiple expectation equations. In the baseline implementation, these objects may be recomputed many times within the same iteration even when the underlying information set is unchanged. Our implementation stores such intermediate objects once per iteration and reuses them whenever the same inputs reappear. This avoids repeated estimation of identical auxiliary forecasting objects and therefore substantially reduces the cost of each iteration.

Third, the evaluation of conditional expectations contains a component that depends only on the agents' information structure and a component that depends on the particular variable being forecast. The baseline implementation largely recomputes both parts across forecasting equations. We instead separate these components, compute the common signal-extraction objects once, and reuse them across all expectations formed under the same information set. We also precompute fixed objects associated with the polynomial approximation outside the innermost loop. These changes reduce repeated transform and linear-algebra operations and are especially important in models with endogenous information, where expectations must be recomputed many times during solution.

Table B.1 shows that, under a common fixed-step configuration with $dft = 512$, $crit = 10^{-6}$, $nit = \{10, 10000, 0\}$, $step = 0.25$, and $arma = \{6, 6, \texttt{false}\}$, these refinements reduce computation time by a factor of between 1.9 and 6.7 across standard examples, with a geometric-mean speedup of 2.6.

Table B.1: Computation time: baseline versus improved implementation

Example	Baseline (sec.)	Our Algorithm (sec.)	Speedup
`Ex2a_dsge_mp`	6.54	3.42	1.91
`Ex2b_dsge_fp`	45.35	6.80	6.67
`Ex3a_hank_sym`	76.19	40.04	1.90
`Ex5b_noisy_price`	2.92	1.54	1.90

Notes: Entries report mean solution times over three runs. All runs converged. Both implementations use the same configuration: $dft = 512$, $crit = 10^{-6}$, $nit = \{10, 10000, 0\}$, $step = 0.25$, and $arma = \{6, 6, \texttt{false}\}$. The improved implementation additionally uses safeguarded acceleration with parameters $\{4, 5, 10^{-10}, 1.25\}$. The baseline implementation does not support this acceleration step.

B.3 Autocovariances of data and model

Table B.2: Pre-1990 Covariance and Autocovariance Moment Fit

	Data				*Model*			
	y_t	π_t	$E_t y_t$	$E_t \pi_t$	y_t	π_t	$E_t y_t$	$E_t \pi_t$
Lag 0								
y_t	8.93	-4.38	3.86	-2.12	8.93	-3.46	4.24	-3.07
π_t	-4.38	5.57	-2.61	3.51	-3.46	5.11	-3.07	4.00
$E_t y_t$	3.86	-2.61	2.90	-1.54	4.24	-3.07	2.90	-2.70
$E_t \pi_t$	-2.12	3.51	-1.54	3.39	-3.07	4.00	-2.70	3.39
Lag 1								
y_t	7.82	-4.63	2.99	-1.80	7.42	-3.20	3.54	-2.72
π_t	-3.65	5.33	-2.29	3.16	-2.75	4.49	-2.55	3.42
$E_t y_t$	4.12	-2.82	2.45	-1.36	4.28	-3.15	2.74	-2.61
$E_t \pi_t$	-2.10	3.75	-1.48	3.20	-2.83	4.01	-2.46	3.20
Lag 2								
y_t	5.77	-4.41	2.17	-1.40	5.86	-2.79	2.87	-2.30
π_t	-2.65	4.85	-1.85	2.79	-2.08	3.77	-2.05	2.82
$E_t y_t$	3.80	-2.82	1.92	-1.12	3.86	-2.90	2.39	-2.31
$E_t \pi_t$	-2.01	3.84	-1.46	2.99	-2.29	3.64	-2.07	2.78
Lag 3								
y_t	3.15	-3.83	1.36	-0.97	4.53	-2.34	2.28	-1.89
π_t	-1.48	4.19	-1.25	2.38	-1.53	3.08	-1.62	2.27
$E_t y_t$	2.85	-2.62	1.36	-0.83	3.24	-2.49	1.99	-1.94
$E_t \pi_t$	-1.79	3.79	-1.44	2.78	-1.70	3.08	-1.66	2.29

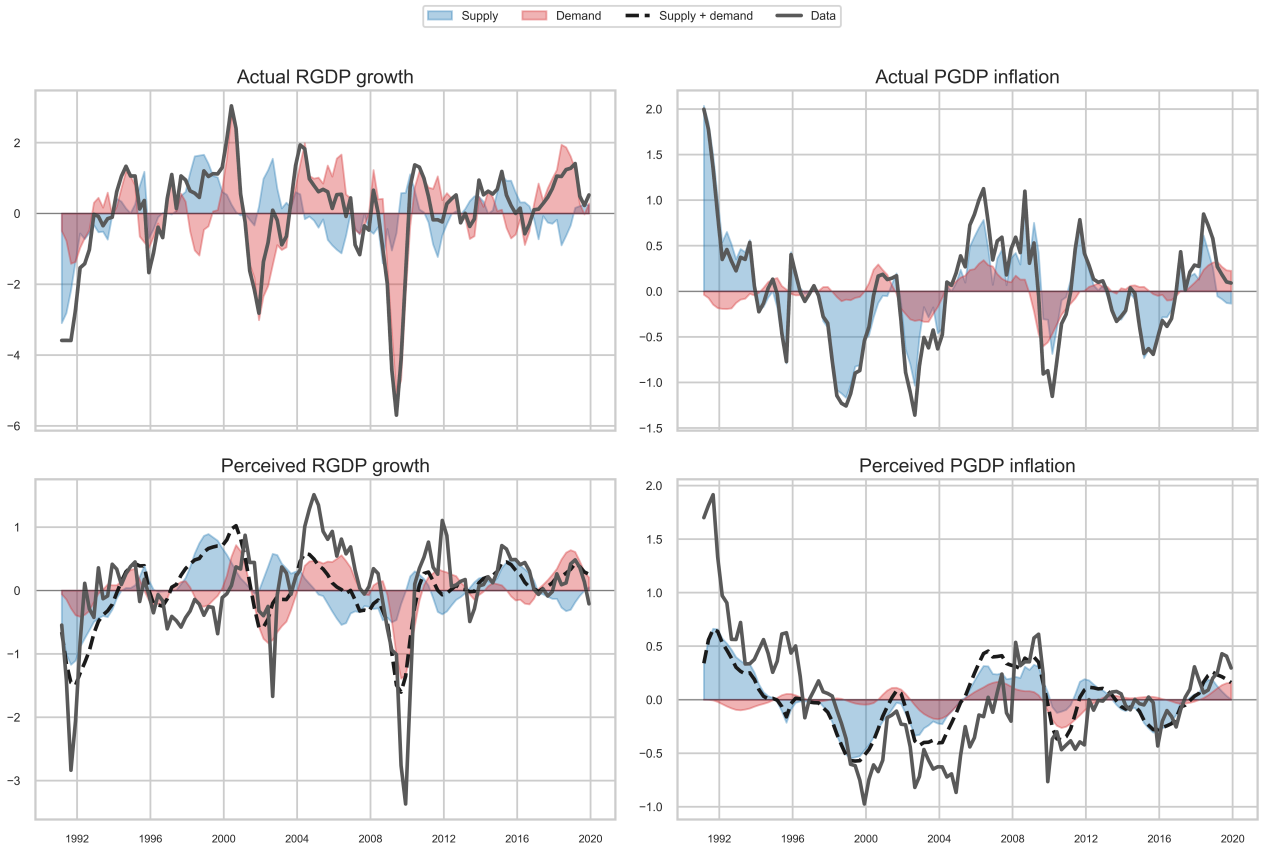
Notes: Entries at lag h are $\text{cov}(x_t, x_{t-h})$. Rows index the current variable x_t and columns index the lagged variable x_{t-h}.

Table B.3: Post-1990 Covariance and Autocovariance Moment Fit

	Data				*Model*			
	y_t	π_t	$E_t y_t$	$E_t \pi_t$	y_t	π_t	$E_t y_t$	$E_t \pi_t$
Lag 0								
y_t	1.93	-0.24	0.54	-0.39	1.93	-0.24	0.69	-0.15
π_t	-0.24	0.37	0.06	0.17	-0.24	0.37	-0.15	0.17
$E_t y_t$	0.54	0.06	0.56	-0.14	0.69	-0.15	0.34	-0.10
$E_t \pi_t$	-0.39	0.17	-0.14	0.27	-0.15	0.17	-0.10	0.10
Lag 1								
y_t	1.59	-0.31	0.29	-0.38	1.62	-0.22	0.57	-0.13
π_t	-0.05	0.31	0.11	0.12	-0.18	0.34	-0.12	0.15
$E_t y_t$	0.73	-0.00	0.43	-0.19	0.74	-0.16	0.33	-0.10
$E_t \pi_t$	-0.31	0.19	-0.09	0.24	-0.14	0.18	-0.09	0.10
Lag 2								
y_t	1.07	-0.34	0.12	-0.32	1.31	-0.21	0.46	-0.12
π_t	0.08	0.24	0.13	0.07	-0.13	0.30	-0.10	0.13
$E_t y_t$	0.74	-0.06	0.28	-0.21	0.72	-0.17	0.30	-0.10
$E_t \pi_t$	-0.20	0.19	-0.06	0.20	-0.13	0.18	-0.08	0.09
Lag 3								
y_t	0.52	-0.34	-0.02	-0.23	1.04	-0.19	0.37	-0.10
π_t	0.14	0.18	0.12	0.02	-0.10	0.27	-0.08	0.12
$E_t y_t$	0.58	-0.08	0.17	-0.17	0.66	-0.16	0.27	-0.09
$E_t \pi_t$	-0.12	0.18	-0.02	0.17	-0.11	0.18	-0.07	0.09

Notes: Entries at lag h are $\text{cov}(x_t, x_{t-h})$. Rows index the current variable x_t and columns index the lagged variable x_{t-h}.

Figure B.1: Historical Decomposition Before 1990



Notes: Stacked-area plots attribute quarterly fluctuations in real GDP, inflation, and their SPF expectations, after 1990, to the structural demand (blue) and supply (red) shocks recovered via model inversion. The first row shows the results for real GDP and its forecast, while the second row shows the results for inflation and its forecast.

B.4 Post-1990 historical decomposition

Figure B.1 reports the historical decomposition for the post-1990 sample. The model misses expectation dynamics at the beginning of the sample. This is partly mechanical: the inversion initializes the model with no pre-sample shocks. Early observations may reflect persistent effects of shocks realized before the sample begins, but these shocks are not included in the ARMA representation used for the inversion. The recovered in-sample shocks therefore partly absorb missing initial conditions. For this reason, we interpret the early-sample decomposition with caution.

The model also misses perceived RGDP growth during 1995–2000. In this period, perceived RGDP growth is negative while realized RGDP growth is positive. This pattern is difficult to explain through the information-friction channels.